PHYS 661 **Homework Set 3** **Due Thursday, February 6, 2025**

Reading assignment: sections 15.3 and 15.6-15.8 and 16.1-16.3 of the text. (Chapters 13 and 14 and sections 15.4 and 15.5 are optional, if you are feeling especially enthusiastic.)

Problem 1. A spinless particle of charge e and mass m is confined to a cubic box of side L with center at the origin. A weak uniform electric field E is applied, with direction parallel to one of the sides of the box, and the electrostatic potential for the field is taken to be zero at the center of the cubic box. [Hint: for a particle confined to a 1-d box, we found the wavefunctions in (6.4.10) and (6.4.11) of the text, depending on whether n is odd or even, respectively. However, for this particular problem, you may find it more convenient to use instead the following equivalent version,

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L} + \frac{n\pi}{2}\right),$$

which is valid for both even and odd n, up to physically irrelevant signs.]

(a) Write down the unperturbed energy eigenvalues and corresponding normalized wavefunctions, in terms of appropriate quantum numbers.

(b) Show explicitly that, to first order in the perturbation E, the ground state energy is unchanged.

(c) Find the change in the ground state energy at second order in E. You should leave your answer in terms of an infinite sum of the form

$$\text{SUM} = \sum_{n=2,4,6,\ldots} \frac{n^p}{(n^2-1)^q}$$

where p and q are certain integers that you will find. Evaluate SUM to 3 significant digits. How does the change in the ground state energy scale with the size of the box L?

Hint: you may find the following definite integrals useful:

$$\int_{-L/2}^{L/2} du\ \sin\left(\frac{n\pi u}{L} + \frac{n\pi}{2}\right) \sin\left(\frac{n'\pi u}{L} + \frac{n'\pi}{2}\right) = \begin{cases} \dfrac{L}{2} & (n = n') \\ 0 & (n \neq n'). \end{cases}$$

and

$$\int_{-L/2}^{L/2} du\, u \sin\left(\frac{n\pi u}{L} + \frac{n\pi}{2}\right) \sin\left(\frac{n'\pi u}{L} + \frac{n'\pi}{2}\right) = \begin{cases} -\dfrac{4L^2}{\pi^2} \dfrac{nn'}{(n^2 - n'^2)^2} & (n + n' = \text{odd}) \\ 0 & (n + n' = \text{even}). \end{cases}$$

Problem 2. Consider the $n = 3$ states of the Hydrogen atom (neglect spin and fine structure effects).

(a) For the unperturbed states $|n, l, m\rangle$ with $n = 3$, find all of the non-zero matrix elements $\langle 3, l', m'|Z|3, l, m\rangle$. (Use the dipole selection rules and hermeticity of the operator Z to reduce the labor.)

(b) Include an electric field $\vec{E} = E\hat{z}$, treated as a perturbation. To first order in E, obtain the energy eigenvalue corrections using degenerate perturbation theory. Give the corresponding energy eigenstates to order E^0, writing your answers in terms of the unperturbed energy eigenstates.

Problem 3. Consider a particle of mass m moving in 1 dimension in the very strongly attractive potential $V(X) = \lambda X^8$. Using a Gaussian trial wavefunction, estimate the ground state energy and wavefunction. What is the corresponding estimate for $\langle X^2\rangle$? Do the same for the first excited state, using a trial wavefunction of the form x multiplied by a Gaussian.