Problem 1. (a) Show that

\[ \int_{-1}^{1} x^n P_l(x) \, dx = 0 \quad \text{for } n < l. \]

[Hint: Use Rodrigues’ formula, and integrate by parts repeatedly.] Use this to show that

\[ \int_{-1}^{1} P_n(x) R_l(x) \, dx = 0 \quad \text{for } n \neq l. \]

(b) Evaluate

\[ \int_{-1}^{1} x^l P_l(x) \, dx \]

in terms of \( l \). [Hint: Use Rodrigues’ formula, and integrate by parts \( l \) times. After a change of variables \( u = x^2 \), the part of the final integral from \( x = 0 \) to \( x = 1 \) can be written in terms of a Beta function, and the integral from \( x = -1 \) to \( x = 0 \) is the same. The resulting Beta function can then be written in terms of Gamma functions as usual, which can then be simplified.]

Problem 2. Use the generating function for Legendre polynomials to evaluate the following:

\[ P_l(0) \quad \text{and} \quad \left. \frac{dP_l(x)}{dx} \right|_{x=0}. \]

[Hint: in each case, use the binomial expansion for \((1 + t^2)^n\).]