Physics 485 **Homework 6** **Due February 27, 2003**

Problem 1. The natural period of vibration of a physical system with damping proportional to the velocity is 5 seconds. If the damping force were removed, the period would become 3 seconds. Find the differential equation of motion of the system and its general solution.

Problem 2. Find the general solutions for the following differential equations:
(a) $y'' - 2y' + y = 2x\cos(x)$ (b) $5y'' + 6y' + 2y = x^2 + 6x$.

Problem 3. The force of gravitational attraction on a mass m a distance r from the center of the Earth ($r > R =$ the radius of the earth), is mgR^2/r^2. The differential equation of motion of a mass m projected radially outward is therefore:

$$m\frac{d^2r}{dt^2} = -mgR^2/r^2.$$

Assume that $v = v_0$ when $r = R$. Find the velocity v as a function of r, and then use that result to find $r(t)$. Find the escape velocity, that is, the smallest v_0 for which the mass can escape to $r = \infty$.

Problem 4. Linear differential equations of any order (meaning any number of derivatives) with constant coefficients can be solved by a method similar to the ones we have used for 2nd-order equations. For example, consider the 3rd-order equation:

$$y''' - 2y'' - y' + 2y = e^{-x}.$$

(a) Solve the homogeneous version of the equation (obtained by replacing e^{-x} by 0 on the right-hand side), by substituting in a guess $e^{\alpha x}$ and finding the auxiliary equation satisfied by α, and then solving it.
(b) Write the equation in the form:

$$(D - \alpha_1)(D - \alpha_2)(D - \alpha_3)y = e^{-x},$$

where $\alpha_{1,2,3}$ are numbers you know from part (a).
(c) Find the general solution to the equation, by solving three linear first-order differential equations in succession. Check your answer by plugging it in to the original equation.