Problem 1. The motion of a falling body in a resisting medium may be described by:

\[ m \frac{dv}{dt} = mg - bv, \]

when the retarding force is proportional to the velocity, \( v \). Find the velocity as a function of time. Evaluate the constant of integration by demanding that \( v(0) = 0 \).

Problem 2. The rate of evaporation from a spherical drop of liquid (constant density) is proportional to its surface area. Assuming this to be the only mechanism of mass loss, write down a differential equation for the radius of the drop as a function of time, and solve it.

Problem 3. Find the general solutions for:

(a) \[ \frac{dy}{dx} = \frac{2xy}{y - x^2} \]

(b) \[ \frac{dy}{dx} = \frac{y}{ey - x} \]

[Hint: for (b), you may find it easiest to solve for \( x \) as a function of \( y \).]

Problem 4. As discussed in class, the differential equation governing the density (number of particles per unit volume) of annihilating dark matter particles in an expanding universe is:

\[ \frac{dn}{dt} + 3H(t)n = -kn^2 + S(t) \]

where \( H(t) = \dot{a}/a \) is known as the “Hubble constant” (although it is not really constant). Assume that the source term \( S(t) \) is negligible (this is not a completely realistic assumption). The result is a special case of Bernoulli’s equation.

(a) Suppose that the scale factor \( a(t) \) is given by \( a_0(t/t_0)^{2/3} \). (This is the case for a universe in which the expansion rate is said to be “matter dominated”.) Convert the above equation into a linear equation, find an integrating function, and find the general solution. How does the solution behave for very large \( t \)? In that limit, which is more important in determining the density \( n(t) \), the expansion of the universe, or the annihilation? Answer the same questions for very small \( t \).

(b) Repeat part (a) if the scale factor is \( a(t) = a_0(t/t_0)^{1/2} \) and again if \( a(t) = a_0e^{H_0t} \). (These two cases correspond to universes in which the expansion rate is said to be “radiation dominated” and “vacuum energy dominated”, respectively.)