Physics 485 **Midterm** **Due 5:00 PM, May 8, 2003**

Rules: You may consult your class notes and any published books that you want. **You are on your honor not to consult with each other, or with people outside the class, and not to use a computer.** You can reach me at `spmartin@niu.edu` if you have any questions. I may give out hints if you are stuck. **Show your work.**

Problem 1. (25 points) Find the Fourier transform of the "triangular pulse function" of height h and width $2a$:

$$f(x) = \begin{cases} (1 - |x|/a)h, & (|x| \leq a) \\ 0, & (|x| \geq a) \end{cases}$$

For $h = 1/a$ and $a \to \infty$, $f(x)$ approaches the delta function $\delta(x)$. Show that its Fourier transform approaches the Fourier transform of $\delta(x)$ (namely a constant) in that limit.

Problem 2. (25 points) Consider a drum head in the shape of a sector of a circle of radius a and angle β:

The speed of waves travelling on the drumhead is v.
(a) For $\beta = \pi/2$, what are the lowest two frequencies?
(b) What are all of the frequencies of oscillation, for general β?

Problem 3. (25 points) Consider a quantum-mechanical particle free to move within a right circular cylindrical box of radius a and height h. It obeys the Schrodinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi - E\psi = 0$$

and the wavefunction ψ vanishes on the walls and ends of the cylinder. What are all of the energy eigenvalues?

Problem 4. (25 points) Assume that the density of free neutrons $n(\vec{\mathbf{r}}, t)$ in U_{235} obeys the differential equation:

$$\nabla^2 n + \lambda n = \frac{1}{\kappa}\frac{\partial n}{\partial t}$$

where λ and κ are fixed positive constants. Consider a sphere of U_{235} with radius R, with the boundary condition that the neutron density vanishes on the surface:

$$n(\vec{\mathbf{r}}, t)\Big|_{r=R} = 0.$$

We want to look for solutions of the form:

$$n(\vec{\mathbf{r}}, t) = N(\vec{\mathbf{r}})e^{Ct}.$$

If $C \leq 0$, this represents a stable solution, but if $C > 0$, then an exponential instability results.

(a) Use separation of variables $N(\vec{\mathbf{r}}) = R(r)Y_\ell^m(\theta, \phi)$, and find all of the allowed values of C, in terms of κ, λ, and R and dimensionless numbers.

(b) For fixed κ, λ, and R, what is the largest allowed value of C?

(c) What is the critical radius of the sphere, R_c, such that if $R > R_c$, then there will be a least one allowed positive value of C? (An explosion will ensue. Don't try this at home.)

(d) Explain how you could have guessed the answer to part (c), up to a multiplicative factor, using dimensional analysis and almost no work. (Hint: what are the dimensions of λ and κ?)