Physics 470 **Homework 7** **Due Monday, October 23, 2017**

Reading assignment: Section 9.3 of Griffiths

Problem 1. Consider the following electric field in *spherical* coordinates:

$$\vec{\mathbf{E}}(r,\theta,\phi,t) = E_0 \frac{\sin\theta}{kr}\left[\cos(kr-\omega t) - \frac{1}{kr}\sin(kr - wt)\right]\widehat{\phi}$$

with $k = \omega/c$. This is the electric field for the simplest kind of spherical electromagnetic wave emanating from the point $r = 0$. [Note that the appearance of $kr - \omega t$ means it is an outgoing wave, but it does not maintain constant shape as it moves, instead getting smaller for large r because of the $1/kr$ factors.] In your calculations, you will want to use the notation $kr - \omega t \equiv \Phi$ for convenience.

(a) Show that this electric field satisfies Gauss' law in a vacuum for all $r > 0$.

(b) Compute the curl of $\vec{\mathbf{E}}$. Then, by requiring that Faraday's law is satisfied, find $\vec{\mathbf{B}}$. This will require you to integrate with respect to t. You will find the formulas for the indefinite integrals

$$\int \cos\Phi\, dt = -\frac{1}{\omega}\sin\Phi \qquad \text{and} \qquad \int \sin\Phi\, dt = \frac{1}{\omega}\cos\Phi$$

to be useful for this. You should find

$$\vec{\mathbf{B}} = E_0\cos\theta\left(\frac{x_1}{r^2}\sin\Phi + \frac{x_2}{r^3}\cos\Phi\right)\widehat{r} + E_0\sin\theta\left(\frac{x_3}{r}\cos\Phi + \frac{x_4}{r^3}\cos\Phi + \frac{x_5}{r^2}\sin\Phi\right)\widehat{\theta}$$

where x_1, x_2, x_3, x_4, x_5 are quantities (dependent only on ω, k) that you will determine.

(c) Show explicitly that the remaining two of Maxwell's equations are satisfied by the $\vec{\mathbf{E}}$ and the $\vec{\mathbf{B}}$ that you found.

(d) Calculate the Poynting vector $\vec{\mathbf{S}}$. Average $\vec{\mathbf{S}}$ over a full cycle to get the intensity vector $\vec{\mathbf{I}}$. (The vector $\vec{\mathbf{I}}$ should point in the radial direction and has a radial dependence exactly proportional to $1/r^2$, even though $\vec{\mathbf{S}}$ does not have these properties.)

(e) Integrate $\vec{\mathbf{I}} \cdot d\vec{\mathbf{a}}$ over a spherical surface centered at the origin to determine the total power radiated. [As a check, this must be independent of the size of the sphere you choose to integrate over, of course.]

Problem 2. A linearly polarized electromagnetic plane wave is propagating in a non-conducting linear medium with permeability $\mu = \mu_0$. The magnetic field of the wave is given by

$$\vec{\mathbf{B}} = B_0 \sin(\frac{2\omega}{c}x + ay + \omega t)\ (b\hat{x} + \hat{y})$$

where a and b are non-negative constants.

(a) Find a and b.

(b) What is the wavevector of the wave? [Hint: be careful with the sign.]

(c) What are the index of refraction and the permittivity of the material?

(d) Find the electric field $\vec{\mathbf{E}}$ and the Poynting vector of the wave.

Now suppose that the material fills the region of space $x > 0$, while the region of space $x < 0$ is filled with vacuum. The wave given above is incident on the boundary between the two regions at the plane $x = 0$.

(e) Find the electric and magnetic fields of the reflected wave (in the region $x > 0$).

(f) Find the electric and magnetic fields of the transmitted wave (in the region $x < 0$).

Problem 3. The Voice of America can sometimes be heard in the radio shortwave band at a frequency of 17725 kHz. (Actually the frequency depends on the time of day, language, and the location of the transmitter.) TV station KSBW Channel 8 in Salinas, California (the lettuce capital of the world) broadcasts in the VHF band at 183 MHz. FM radio station KROQ in Pasadena broadcasts at a frequency of 106.7 MHz. In Chicago, ESPN radio broadcasts in the AM band at 1000 kHz. If you are reasonably far from the transmission tower, then the electromagnetic signal in each case is almost a plane wave. Compute the angular frequency (ω), period, wavenumber and wavelength for each case, and make a table of your findings, for the the four frequencies listed above. Be sure to specify your units.

Problem 4. The total electricity consumption in the United States each year is about 4×10^{12} kiloWatt-hours. It is proposed to provide this with a giant array of solar cell panels, each 1 meter square, pivoted so that each always points towards the Sun. Estimate the number of solar panels that would be needed. Assume the efficiency of a solar cell is about 20%, and make a very rough justifiable estimate of the average intensity of sunlight over the course of a year. (Don't forget that night happens. . .)