

Reading: Griffiths pages 94-107 and 113-118.

Problem 1 At least one of these is an impossible electric field in electrostatics. Which one(s) and why? (Each of  $\kappa_{1,2,3}$  is some constant with appropriately chosen units.)

(a)  $\vec{\mathbf{E}} = \kappa_1 [2xy \hat{x} + (x^2 + 2yz)\hat{y} + y^2 \hat{z}]$  (rectangular coordinates)

(b)  $\vec{\mathbf{E}} = \kappa_2 [\cos \phi \hat{r} - \sin \theta \sin \phi \hat{\phi}]$  (spherical coordinates)

(c)  $\vec{\mathbf{E}} = \kappa_3 [z \cos \phi \hat{r} + r \cos \phi \hat{z} - z \sin \phi \hat{\phi}]$  (cylindrical coordinates)

Problem 2 A solid sphere of radius  $R$  carries charge density  $\rho(r) = \rho_0 r^2 / R^2$ , where  $r$  is the distance to the center and  $\rho_0$  is a constant. Show that the energy stored in the configuration is

$$W = X\pi\rho_0^2 R^5 / \epsilon_0$$

where  $X$  is a rational number that you will compute. [Hint: first find the electric field both inside and outside of the sphere, using Gauss' Law in integral form.]

Problem 3 A capacitor consists of two parallel square metal plates of side 1 cm. They are 0.2 mm apart.

(a) What is the capacitance, numerically, in pF? Remember that  $1 \text{ pF} = 10^{-12} \text{ F}$ .

(b) If the two plates are connected to the terminals of a 9 volt battery, what is the magnitude of the charge on each plate, numerically, in Coulombs?

(c) How much energy, in Joules, is stored in the capacitor in part (b)?

Problem 4 Consider two long coaxial metal cylindrical tubes of length  $L$ , with radii  $a$  and  $b$ . [See Griffiths Figure 2.53, p. 107 (4th edition) or p. 106 (3rd edition).]

(a) Compute the electric field and the potential everywhere for  $a < r < b$ , if the inner cylinder is given a charge  $+Q$  and the outer cylinder a charge  $-Q$ , and assuming  $L \gg a, b$ .

(b) Show that the capacitance per unit length is  $K\epsilon_0 / \ln(b/a)$ , where  $K$  is a certain number that you will compute. [Hint: The total capacitance will be proportional to  $L$ , but we are looking for the capacitance per unit length which does not depend on  $L$ .]

(c) For  $a = 1 \text{ mm}$  and  $b = 3 \text{ mm}$ , what is the result of part (b) numerically in pF/meter?