NIU Ph.D. Candidacy Examination Spring 2018 (1/9/2018)

Electricity and Magnetism

You may solve ALL FOUR problems if you choose. The points of the best three problems will be counted towards your final score of this part of the examination. (40 points each. Total of 120 points)

Do not just quote a result. Show your work clearly step by step.

1. [40 points] **Particle refraction at the boundary between two regions:** We consider a charged particle with mass m, charge q, and total energy E, crossing the boundary between two regions (1) and (2) subjected to different electrostatic potentials Φ_1 and Φ_2 taken to be constants; see Figure 1. The particle's trajectory is taken to be in the (x, y) plane and crosses the boundary defined by the plane $x = 0$.

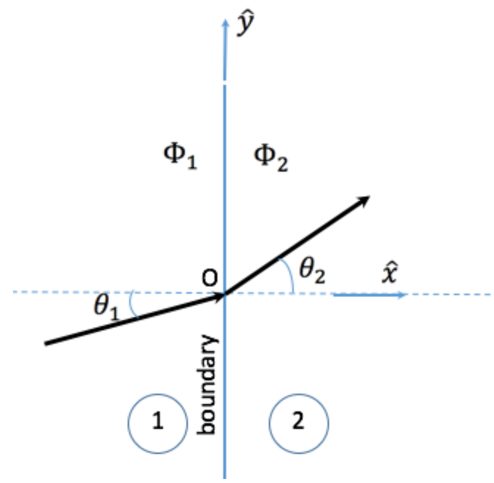

Figure 1

   (a) Given the geometry of the problem, give two conserved quantities. [5 points]

   (b) From (a), show that the particle angles θ_1 and θ_2 are related by a condition similar to Snell's law in Optics. Identify the quantities that play the role of the index of refraction and relate them to the particle properties and potentials Φ_1 and Φ_2. [15 points]

   (c) Discuss the evolution of θ_2 (for $\theta_1 \neq 0$) as a function of the potential difference $\Delta\Phi = \Phi_1 - \Phi_2$ identifying the different possible cases. Especially, give a condition between the angle θ_1, $\Delta\Phi$, and the incident momentum p_1 to have a "grazing" refraction ($\theta_2 = \frac{\pi}{2}$). [13 points]

   (d) Discuss how the boundary could be practically implemented, and what is the needed surface-charge density to establish the needed potential difference. [7 points]

2. [40 points] **Electric multipole expansion**
The potential of an arbitrary localized charge distribution can be expressed in the following electric multipole expansion in powers of $1/r$.

$$V(r)=\frac{1}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{(n+1)}}\int(r')^n P_n(\cos\theta')\rho(r')d\tau' .$$

Figure 2 defines the appropriate variables: θ' is the angle between **r** and **r**'. $d\tau'$ is an infinitesimal volume element. s is the distance between $d\tau'$ and P. $P_n(\cos\theta')$ is the Legendre polynomials in terms of $\cos\theta'$. $P_n(x)$ in general is defined by

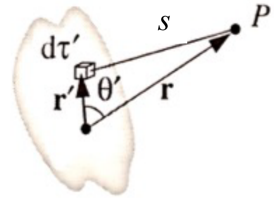

Figure 2

$$P_n(x)=\frac{1}{2^n n!}\left(\frac{d}{dx}\right)^n\left(x^2-1\right)^n .$$

A sphere of radius R, centered at the origin, carries the charge density

$$\rho(r,\theta)=k\frac{R}{r^2}(R-2r)\sin\theta ,$$

where k is a constant, and r, θ are the standard spherical coordinates. Using the above multipole expansion of V in powers of $1/r$, calculate
(a) the monopole term, [10 points]
(b) dipole term, [10 points]
(c) quadruploe term. [10 points]
Then,
(d) find the approximate potential (the lowest non-zero term) for points on the z axis, far from the sphere, and briefly comment on the meaning of the successive terms. [10 points]

3. [40 points] **Moving loop in a constant magnetic field:** A circular loop with initial radius r_0 is inside a solenoid with constant magnetic field $\boldsymbol{B}_0$; see Figure 3. The loop is connected to a bulb and the whole circuit has a resistance R. At time $t = 0$ the loop is moved by pulling the two ends through a hole in the solenoid at constant speed u maintaining the circular shape of the loop (the whole loop remains inside the solenoid).

   (a) What is the current in the loop $I(t)$ and the power $P(t)$ dissipated in the circuit? [12 points]
   (b) What is the total energy dissipated in the circuit? [8 points]
   (c) What is the force one should apply at point A to pull the wires at speed u? [10 points]
   (d) Show that the work done mechanically equals the energy dissipated in the lamp circuit computed in question (b). [10 points]

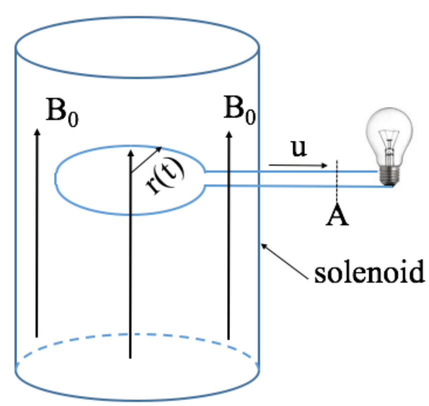

Figure 3

4. [40 points] An infinitely long, non-magnetic, solid cylinder has radius a. Consider a finite co-axial cylinder of smaller radius r and length ℓ. For each of the independent situations described in parts (a) and (b), find the time derivative of the total electromagnetic field energy inside, and the flux of energy through the surface, for the finite cylinder. If they do not agree, provide a reason and account for the difference quantitatively.

   (a) The infinite cylinder of radius a is wrapped with n turns of wire per unit length along the axis, carrying a slowly varying current $I(t) = I_0 + kt$, where k is a constant. Work to first order in k, dropping all quantities of order k^2. [20 points]

   (b) The infinite solid cylinder is not wrapped with wire, but is made from material with finite conductivity σ, and carries a current I along its axis of symmetry, with magnitude that is uniform (in space) and constant (in time). [20 points]