3.1 Typos in the Lecture Notes

Read the lecture notes carefully. Unfortunately there are still typos in it. You will get a prize for each typo you find!

3.2 Group $\mathcal{P}_3$

- Verify that the two-dimensional representation $\Gamma_3$ of the permutation group $\mathcal{P}_3$ is, indeed, irreducible.
- Verify the multiplication table for the $\mathcal{P}_3$. 
3.3 Clebsch-Gordan coefficients for the group $P_3$

This is a tricky problem!

We say that a group $G$ is simply reducible if its product representations $\Gamma_I \times \Gamma_J$ contains the IRs $\Gamma_K$ only with multiplicities $a^{IJ}_K = 0$ or $1$. For such groups the CGC can be derived from the representation matrices of the IRs.

- Starting from theorem 11, use the orthogonality relation for the representation matrices in order to derive an equation, where the CGCs appear on one side, the known representation matrices appear on the other side.

- Assuming that $G$ is simply reducible, use this relation to define the value of one nonzero CGC for fixed $I, J, K$ (e.g., by choosing the phase convention that this CGC will be positive).

- Sketch how you can derive the remaining CGC for the given values of $I, J, K$.

- The group $P_3$ is simply reducible. Confirm the CGCs given in class (up to a global phase for the CGCs for given $I, J, K$).