## 1.1 Implications of the Group Axioms

Let $a$ be an arbitrary group element. Show

- $a^{-1}\,a = a\,a^{-1} = e$
- $a\,e = e\,a = a$
- There is exactly one identity element with $e^{-1} = e$.
- Every group element has exactly one inverse.

## 1.2 Group with Three Elements

Show by setting up the group multiplication table that there is only one group with three elements.

# 1.3 Symmetry Group of an Equilateral Triangle

- Identify all symmetry operations (rotations, reflections of an equilateral triangle.
- Set up the group multiplication table for this symmetry group.
  In Koster's notation, this group is called $C_{3v}$.

# 1.4 Generators of a Group

An alternative characterization of a group is based on its *generators*. These are elements of the group satisfying certain characteristic generating relations. Starting from these relations we can obtain all elements of the group as products of the generators.
Here we want to study a (noncommutative) group with two generators $p$ and $q$ satisfying the generating relations

$$p^3 = e \qquad q^2 = e \qquad (qp)^2 = e\,.$$

- Show: the group contains six elements $\{e, p, p^2, q, qp, qp^2\}$.
- Set up the group multiplication table. Compare with the previous problem.