S-Plane Bode Plots, Identifying Poles and Zeros in a Circuit Transfer Function

Reza Hashemian
Northern Illinois University
reza@niu.edu
Outline

• Extraction of roots (poles and zeros) from the transfer function polynomials
• Bode plots for three types of roots: on the $\sigma$ axis, complex conjugate roots, and roots on the $j\omega$ axis
• Sweeping the $j\omega$ axis along the $\sigma$ axis in the s-plane
• Root extraction using Bode plots
• Simulation examples
• Conclusion
Extraction of roots (poles and zeros) from a transfer function polynomials

- A circuit transfer function as generated through a symbolic representation

\[ T(s) = \frac{N(s)}{D(s)} = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots + a_0}{s^n + b_{n-1} s^{n-1} + \ldots + b_0} \]

- Solution to the transfer function polynomials, identifying the poles and zeros

\[ T(s) = \frac{N(s)}{D(s)} = H \frac{\prod_{i=1}^{r} (1 + s / z_i) \prod_{k=1}^{p} (1 + a_k s + b_k s^2)}{\prod_{j=1}^{s} (1 + s / p_j) \prod_{l=1}^{q} (1 + a_l s + b_l s^2)} \]
Types of poles and zeros in a transfer function

• We categorize poles and zeros (roots) in a transfer function into three types:

  • Roots on the $\sigma$ axis:
    Regular RL and RC circuits have roots on the $\sigma$ axis

  • Complex conjugate roots:
    Typical RLC circuits, and RL and RC circuits with feedback generally have their transfer functions with complex roots

  • Roots on the $j\omega$ axis:
    Oscillators, LC and tuned amplifiers have their roots on the $j\omega$ axis by design
Bode plots for transfer functions with roots in the three specified regions

\[ T(s) = \frac{10^4 + 1010s + s^2}{10^{-3} + 100s + 101s^2 + s^3} \]  

\[ T(s) = \frac{1 + 0.1s + 0.01s^2}{1 + 0.8s + s^2} \]  

\[ T(s) = \frac{1 + 0.01s^2}{1 + s^2} \]  

- **Real roots**
- **Complex conjugates**
- **Imaginary roots**
Extraction of roots (poles and zeros) from transfer function Bode-plots, a reverse process
Methods to extract roots in all three categories

• As we can see, the $J\omega$ axis roots are the simples roots to detect and extract. The problem is then how to bring the other two kinds of roots on the $J\omega$ axis.

• For the complex conjugate roots we can shift the $J\omega$ axis along the real axis until the roots intercept with the $J\omega$ axis.

• For the real axis roots, however, we can choose one of the two following ways to put them on the $J\omega$ axis.
  • Use the same method applied to the complex conjugate roots.
  • Turn the real axis virtually 90 degrees to place the real roots on the $J\omega$ axis.

The latter method is covered in a published article and will not be presented here.
The $j\omega$ axis transformation along the $\sigma$ axis and its mapping on the circuit
Application of the $j\omega$ axis transformation in simple RC circuits

The $j\omega$ axis transformation used to change a high pass filter to an allpass one.

Sequence of steps moving the $j\omega$ axis along the $\sigma$ axis for the allpass filter.
Sweeping the $j\omega$ axis along the $\sigma$ axis in the $s$-plane in an allpass filter

\[
T(s) = \frac{V_o(s)}{V_i(s)} = \frac{s^2 - s \frac{\omega_0}{Q} + \omega_0^2}{s^2 + s \frac{\omega_0}{Q} + \omega_0^2}
\]

A second order allpass filter; (a) the original filter; (b) The filter with added resistors for the $j\omega$ transformation.
Application of the $j\omega$ axis transformation in amplifier bandwidth design

An amplifier small signal equivalent circuit with feedback; (a) the original amplifier circuit; (b) the modified circuit for a stable response.
Pole extraction in a bridge T-Coil circuit for wideband amplifier

The bridge T-Coil circuit for wideband amplifier

The bridge T-Coil circuit amplifier transfer functions. A series of the transfer function Bode plots shown as the $j\omega$ axis moves along the $\sigma$ axis.
Extraction of zeros in a band-pass analog filter.

A simplified and linearized band-pass analog filter.

A series of the transfer function Bode plots for the band-pass filter. Plots shown as the $j\omega$ axis moves along the $\sigma$ axis.
Extraction of poles in a band-pass analog filter

Circuit with two pairs of complex conjugate poles and two pairs of complex conjugate zeros

Sequence of six Bode plots for the circuit to determine the poles locations.
Conclusion

• A new procedure, called s-plane Bode plots, is introduced to virtually move the $j\omega$ axis along the real axis in steps.
• In each step a Bode plot (magnitude or phase angle, or both) is generated, and overall a plane of Bode plots are generated.
• In the process poles and zeros of a transfer function will fall on the moving $j\omega$ axis, allowing the roots to be identified and extracted.
• Application of the s-plane Bode plots extends to other areas such as designs for bandwidth, frequency response adjustments, circuit stability, and control.
• Another application of the procedure is to generate 3D Bode plots that cover the entire S-Plane.
Thank you!