Modern and Statistical Physics

Do 3 out of 5 problems

Problem 1: (40 points)

(a) The equation of state of a van der Waals gas is

\[
(P + \frac{a}{v^2})(v - b) = RT
\]

where \(P\) is pressure, \(T\) is temperature, \(R\) is gas constant, \(v\) specific volume, \(a\) and \(b\) are characteristic constants for a given gas. Qualitatively explain the meaning of the terms \(\frac{a}{v^2}\), and \(-b\). [10 points]

(b) A van der Waals gas undergoes an isothermal expansion from specific volume \(v_1\) to specific volume \(v_2\). Calculate the change in the specific Helmholtz function. [15 points]

(c) Calculate the change in the specific internal energy in terms of \(v_1\) and \(v_2\). [15 points]
Problem 2: (40 points)

A negatively charged muon stops in aluminum, which has Z=13, where it is captured and decays down to the 1S state.

(a) What is the energy of the gamma emitted when the muon drops from the $n = 2$ to the $n = 1$ atomic state? Give your answer in terms of the ground state (1S) energy of hydrogen. [20 points]

If the muon then undergoes the transition: $\mu^- + Al \rightarrow e^- + Al$

(b) What normally conserved quantity is violated? [5 points]

(c) What is the total kinetic energy of the electron in terms of the masses of the muon, electron, and aluminum? [15 points]

masses: muon 105 MeV, electron 0.5 MeV, Aluminum 25000 MeV

Problem 3: (40 points)

One-dimensional relativistic gas: Here we consider a non-interacting gas of $N$ relativistic particles in one dimension. The gas is confined in a container of length $L$, i.e., the coordinate of each particle is limited to $0 \leq q_i \leq L$. The energy of the $i^{th}$ particle is given by $\varepsilon_i = c|p_i|$. 

(a) Calculate the single particle partition function $Z_1(T,L)$ for given energy $E$ and particle number $N$. [12 points]

(b) Calculate the average energy $E_1$ and the heat capacity $C_1$ per particle from $Z_1(T,L)$. [12 points]

(c) Calculate the Boltzmann entropy $S_B(E,N)$ of all $N$ particles. Consider them as indistinguishable. [16 points]

*Hint:* Use Stirling’s formula for large $N$: $N! \approx \sqrt{2\pi N}N^NE^{-N}$. 
Problem 4: (40 points)

A cylindrical resistor has a radius $b$, length $L$, and conductivity $\sigma_1$. At the center of the resistor is a defect consisting of a small sphere of radius $a$ inside which the conductivity is $\sigma_2$. The input and output currents are distributed uniformly across the flat ends of the resistor.

(a) What is the resistance of the resistor if $\sigma_1 = \sigma_2$? [5 points]

(b) Approximate the spherical defect as:
   - a cylinder of radius $a$ and length $a$.
   - concentric with the cylinder of the physical resistor
   - and centered lengthwise on the center of the physical resistor.

   What is the resistance of the resistor if $\sigma_1 \neq \sigma_2$? [25 points]

(c) *Estimate* the relative change in the resistance to first order in $\sigma_1 - \sigma_2$ if $\sigma_1 \neq \sigma_2$. (Make any assumptions needed to simplify your method of estimation). [10 points]
Problem 5: (40 points)

Estimate the following quantities, indicating how you arrived at your estimate (do not try to calculate exact numbers—make only estimates):

Choose only 4 of the following 7 quantities: [10 points each]

(a) the frequency of radiation used in a household microwave oven,

(b) the energy yield of a fission bomb with a 30 kg uranium core,

(c) the energy of impact of the earth with a meteorite 10 meters in diameter,

(d) the temperature of the sun,

(e) the temperature of a 60-watt light bulb filament,

(f) the speed of sound in helium gas at STP,

(g) the total length of blood capillaries in the human body

You may find the following useful:

Speed of sound in a gas: \( v = \left( \frac{\gamma P}{\rho} \right)^{1/2} \), where \( \gamma = \frac{C_P}{C_V} \)

The speed of sound in air is about 330 m/sec

Stefan’s Law: \( F = \sigma_B T^4 \), where \( \sigma_B \approx 6 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4 \)

Wein’s displacement Law: \( \lambda_{max} T \approx 2.9 \times 10^{-3} \text{ m} \cdot \text{K} \)

The average human has about 4 liters of blood

Typical energy release in a fission process is about 200 MeV

Earth’s orbit has radius of about 8 light minutes

The radius of the sun is about 2 light seconds

The earth’s orbital velocity is about 30 km/sec

The sun’s energy flux at the earth’s surface is about 1.3 kW/m²