Do 3 out of 5 problems

1. [40 points] Classical thermodynamics, Entropy of the van der Waals gas.
   a) Express the combined first and second law of thermodynamics. [5 points]
   b) Derive a $Tds$ equation under the isochoric condition. Use specific heat under the isochoric (constant volume) condition, $c_v$. [20 points]
   c) Calculate the entropy for a van der Waals gas, using the above $Tds$ equation. Here, the equation of state of the van der Waals gas is $\left( P + \frac{a}{v^2} \right) (v-b) = RT$. [15 points]

2. [40 points] Statistics of One-dimensional lattice vibration
   There is a one-dimensional lattice with lattice constant $a$ as shown in figure below. An atom transits from a site to a nearest neighbor site every $\tau$ seconds. The probabilities of transiting to the right and left are $p$ and $q = 1 - p$, respectively.

   ![Lattice Diagram](image)

   a) Calculate the average position $\bar{x}$ of the atom at the time $t = N\tau$, where $N \gg 1$. [20 points]
   b) Calculate the mean-square value $(x - \bar{x})^2$ at the time $t$. [20 points]

3. [40 points] Special relativity, a rocketship travelling back home.
   A rocketship is traveling towards the Earth at 0.6c. It fires a projectile of mass $M$ with a velocity of 0.8c in the rocketship's frame directly at the Earth.
   a) What is the energy of the projectile in the Earth's frame?
   
   The rocketship is now traveling perpendicular to a line from the rocketship to the Earth at 0.6c and again fires a projectile at 0.8c in the rocketship's frame and at 90 degrees from the direction of the rocketship's motion (in the spaceship's frame).
   b) What is the energy of the projectile in the Earth's frame?
   c) What is the tangent of the angle of the projectile relative to the rocketship's motion in the Earth’s frame?

   You can ignore the effects of gravity as the rocketship is 2 light years away from the Earth.
4. [40 points] **Higgs boson.**
   a) Show that the process \( \gamma \rightarrow e^+e^- \) is forbidden in free space. [10 points]
   b) Deduce an expression for the energy of a \( \gamma \) from the decay of a Higgs boson: \( H \rightarrow \gamma \gamma \)
in terms of the Higgs boson mass (\( m_H \)), the Higgs boson energy in the lab frame (\( E \)), the speed of the Higgs boson (\( \beta \)), and the emission angle (\( \theta^* \)) of one of the photons in the Higgs boson rest frame. [20 points]
   c) The Higgs boson does not couple directly to photons. Draw the most important, lowest order Feynman diagram that illustrates the decay of the Higgs boson into two photons. Remember to label all the particles in your diagram. [10 points]

5. [40 points] **Optics.**
   A blade of grass standing \( y_{grass} \) tall is \( d_A \) cm in front of a thin positive (convex) lens, \( A \), having a focal length, \( f_A \). A thin negative (concave) lens, \( b \), with a focal length of \( -f_B \) is placed \( d_B \) behind the first lens \( B \). The blade of grass is illuminated by a monochromatic plane wave parallel to the optic axis of this combined lens system.
   a) Draw the ray diagram of imaging the above object by this combined lens system. [10 points]
   b) Where does the first lens, \( A \), form the image of the blade of grass? Express it with the above given parameters. What’s its magnification? [5 x 3 = 15 points]
   c) Where is the final image formed by this combined lens system? Express it with respect to the location of the lens \( B \). [10 points]
   d) What is the total magnification of this combined lens system? [5 points]