
1. 
   a. (20) Show using conservation of energy and momentum that it is not possible for a free electron moving through vacuum to emit a photon.
   b. (20) Now consider the related problem of an electron moving through superfluid helium. Show that in this case the electron can emit a phonon as long as it moves with a velocity exceeding a critical velocity \( v_c \) and find \( v_c \). The excitation spectrum of the phonons in superfluid helium is given by \( E = u p \) where \( u \) is the velocity of sound and \( p \) is the momentum of the phonon. You may do this part in the non-relativistic limit.

2. 
   a. (10) A C-O white dwarf has a radius = \( R \), mass = \( M \) and is assumed to have constant density. Assuming the electrons are degenerate and non-relativistic find the average energy of the electrons in terms of \( M, R \) and fundamental constants.
   b. (10) Repeat part a) assuming the electrons are relativistic.
   c. (10) Assume the radius shrinks by a factor of 2 so \( R' = R / 2 \) while the mass remains the same. Find the relative change in the gravitational and electron energies (assuming both relativistic and non-relativistic) between \( R \) and \( R' \).
   d. (10) Discuss whether the star is more stable (less likely to collapse) if the electrons are relativistic or if they are non-relativistic.

3. When a large number of atoms come together to produce a solid each atomic level broadens into a band. Draw a picture of simplified band structure, define the valence and conduction bands, and describe the temperature-dependent behavior of conductivity for:
   a. (10) Metals (define the term Fermi energy)
   b. (10) Insulators (define the term band gap)
   c. (10) Intrinsic semiconductors
   d. (10) The n-type and p-type semiconductors

4. One mole of a monatomic ideal gas initially at temperature \( T_0 \) expands from volume \( V'_0 \) to \( 2V'_0 \). Calculate the work of expansion and the heat absorbed by the gas for the case of expansion at:
   a. (20) Constant temperature
   b. (20) Constant pressure

5. 
   a. (20) Consider a large number of \( N \) localized particles in an external magnetic field \( H \) (directed along the z direction). Each particle has a spin \( s = 1/2 \). Find the number of states accessible to the system as a function of \( M_z \), the z component of the total spin of the system. Determine the value of \( M_z \) for which the number of states is maximum.
b. (20) Define the absolute zero of the thermodynamic temperature. Explain the meaning of negative absolute temperature, and give a concrete example to show how the negative absolute temperature can be reached.