Stat/Modern/Thermo PhD. candidacy examination. March 2008

Pick 3 questions out of 5.

1) Consider a string of length $L$, mass density $\mu$ and string tension $\tau$. The string is maintained at finite temperature $T$.
   a. Find the average amplitude of a mode of the string of wavelength $\lambda = 2L/n$ in the limit of $k_B T >\gg hf$, with $f$ the frequency of the mode.
   b. Explain why it is necessary to state that $k_B T >\gg hf$.

2) Assume that the crystal lattice structure of solid comprising $N$ atoms can be treated as an assembly of $3N$ distinguishable one-dimensional oscillators (Einstein solid).
   a. What is the partition function $Z$? Use the Einstein temperature $\theta_E (= h\nu/k$, where $\nu$ is natural frequency).
   b. Calculate the Helmholtz function $F$.
   c. Calculate the entropy $S$.
   d. Show that the entropy approaches zero as the temperature goes to absolute zero.

3) The variation of the internal energy $U$ as a function of entropy $S$ is predicted by classical equilibrium thermodynamics to have a functional form of $U - U_0 = \alpha (S - S_0)^n$ where $\alpha$ is a constant for a fixed value of volume.
   a. Show that the temperature as a function of entropy in the case of constant volume is given by $T = \alpha n (S - S_0)^{1/n-1}$.
   b. Show that $C_v = \frac{S - S_0}{n-1} = \frac{1}{n-1} \left( \frac{T}{\alpha n} \right)^{1/n-1}$.

   At some point it may be useful to derive the relationship $\left( \frac{\partial T}{\partial S} \right)_v = \left( \frac{\partial^2 U}{\partial S^2} \right)_v$.

4) Silicon has $Z=14$.
   a. Relative to a hydrogen atom, what are the energy and radius of an electron in the 1s shell?
   b. In the 3p shell?
   c. Assume that there are 2 electrons in the 3p shell. What are the allowed spectroscopic states for this 2-electron system? (Give S, L, J).
   d. What is the energy ordering and why?

5) A charged kaon (at rest) decays by $K^+ \to \pi^+ + \pi^0$ and then $\pi^0 \to \gamma + \gamma$.
   a. In terms of the masses of the particles what is the $\pi^0$’s energy?
   b. What is the maximum photon energy?