1) a) Starting with the first law of thermodynamics and the definition of \( c_p \) and \( c_v \), show that

\[
(c_p - c_v) = p + \left( \frac{\partial U}{\partial V} \right)_T \left( \frac{\partial V}{\partial T} \right)_p .
\]

Here \( c_p \) and \( c_v \) are the specific heat capacities per mole at constant pressure and volume respectively, and \( U \) and \( V \) are the energy and volume of one mole.

b) Use the above result plus the expression

\[
p + \left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial p}{\partial T} \right)_V\]

to find \( c_p - c_v \) for a van der Waals gas with equation of state

\[
(p + \frac{a}{V^2}) (V - b) = RT .
\]

Here \( a \) and \( b \) are constants.

c) Use this result to show that as \( V \to \infty \) at constant \( p \), you obtain the ideal gas result for \( c_p - c_v \).

2) The rotational motion of a diatomic molecule is specified by two angular variables\( \theta \) and \( \phi \) and the corresponding canonical conjugate momenta, \( p_\theta, p_\phi \). Assuming the form of the kinetic energy of the rotational motion to be

\[
\varepsilon_{rot} = \frac{1}{2I} P_\theta^2 + \frac{1}{2I \sin^2(\theta)} P_\phi^2
\]

a) Derive the classical formula for the rotational partition function, \( r(T) \),

\[
r(T) = \frac{2kT}{\hbar^2}
\]

b) Calculate the Helmholtz free energy \( F_{rot} \).

c) Calculate the corresponding entropy and specific heat.

The following may be helpful

\[
\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}
\]

\[
\int \frac{dx}{\sin^2(ax)} = -\cot(ax)/a
\]

3) Assume that the neutron density in a neutron star is 0.1/fm\(^3\) (that is 0.1 neutron per cubic Fermi). Assuming \( T=0 \) and ignoring any gravitational forces calculate the ratio of neutrons to protons to electrons.
Hint: determine their Fermi energy. The electron, neutron and protons masses are 0.511 MeV/c², 939.6 MeV/c² and 938.3 MeV/c². The constant \( hc = 1240 \text{ MeV} \cdot \text{fm} \). You should be able to work out "by hand" an approximate value.

4) A \( \pi - \mu \) atom consists of a pion and a muon bound in a Hydrogen-like atom.

a) What are the energy levels for such an atom compared to those for Hydrogen?

b) \( \pi - \mu \) atoms are produced in \( K_L \) decays (\( K_L \rightarrow \pi - \mu + \nu \)). If the \( K_L \) has \( \beta = 0.8 \) what are the minimum and maximum energies of the \( \pi - \mu \) atom expressed in terms of the \( K \), \( \pi \) and \( \mu \) masses with \( m_e = 0 \)?

c) Approximately what fraction of \( K_L \) decays will produce a \( \pi - \mu \) atom (hint: use the Heisenberg uncertainty principle)?

5) a) You are familiar with the quarter-wave thin film coating that acts as a “reflection-reducer”. For the moment, let us look at a simpler thin film—the air gap between two pieces of glass such as you would find in a Newton’s rings experiment. Why do we get constructive interference in the reflected when the thickness is one-fourth of the wavelength of light or some odd multiple of a quarter wavelength? Why isn’t it constructive at one-half wavelength of the light? For assistance, I present two of the Fresnel equations (in two forms) for reflected light.

\[
\begin{align*}
r_\parallel &= \frac{n_2 \cos \theta_2 - n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} = \frac{\tan(\theta_2 - \theta_1)}{\tan(\theta_1 + \theta_2)} \\
r_\perp &= \frac{n_2 \cos \theta_2 - n_1 \cos \theta_1}{n_1 \cos \theta_1 + n_2 \cos \theta_2} = \frac{-\sin(\theta_2 - \theta_1)}{\sin(\theta_1 + \theta_2)}
\end{align*}
\]

Where the parallel and perpendicular symbols refer to the plane of incidence, and \( i, t \) refer to incident and transmitted media, \( \theta \)'s are angles of incidence and transmission, and \( n \)'s are indices of refraction.

b) In light of the previous, to get destructive reflection in a thin film-i.e.-a quarter-wave film, such as the one illustrated below, what condition must prevail among the indices of refraction for the three media (\( n_0 \) may be taken as \( = 1.0 \) for air.)

c) The destructive interference described in part b) will generally not be complete. Find the value of \( n_2 \) as a function of \( n_1 \) which gives completely destructive interference at normal incidence.
6) In a big-bang theory of the universe, the radiation energy initially confined in a small region adiabatically expands in a spherically symmetric manner. Here the radiation (photon) pressure is expressed as $p = U/V$, and the black body radiation energy density is $u = U/V = aT^4$. The radiation cools down as it expands.

a) Derive a relation between the temperature $T$ and the radius $R$ of the spherical volume of radiation, based purely on thermodynamic considerations.

b) For the above problem, show the total entropy of a photon gas is expressed as $S = \frac{4}{3} aT^3 V$. 