Modern and Statistical Physics

Do 3 out of 5 problems

Problem 1: (40 points)

(a) A laser beam having a diameter $D$ in air strikes a uniform flat piece of glass of index of refraction $n$ at an angle $\theta$. What is the diameter of the beam in the glass?

(b) A narrow beam of white light is incident in air at an angle $\theta$ on a uniform flat sheet of glass of thickness $L$. The index of refraction for red light is $n_r$ and for violet light it is $n_v$. Determine the diameter of the emerging beam assuming the incident beam is exceedingly narrow.

Problem 2: (40 points)

Find the maximum total energy and maximum transverse momentum for the initially at rest particle after the following elastic scatter reactions:

(a) a 10 GeV positron strikes an electron at rest

(b) a 10 GeV proton strikes an electron at rest

(c) a 10 GeV electron strikes a proton at rest

Note: mass electron = 0.5 MeV mass proton=1 GeV (for this problem)
The system above is thermally isolated. It consists of two parts, a bucket (A) containing a mixture of ice and water, and a body (B) with constant heat capacity $C$. The two parts are connected by a thermodynamic device which can extract heat from part (A) and add heat to part (B).

The entire system is initially at temperature $T_0$ (the absolute temperature of melting ice). The latent heat of melting of ice is $L$ per unit mass. Ignore all effects due to changing volumes.

(a) How much heat must be removed from (A) to freeze an additional mass $m$ of water? What is the entropy change of (A) in this process? [10 points]

(b) In this process (B) absorbs the heat released by the thermodynamic device, and its temperature increases to $T_1$. Calculate the heat absorbed $\Delta Q_b$ and the entropy change $\Delta S_b$ of the body (B) in terms of $T_1$. [10 points]

(c) What is the minimum possible $T_1$? What is the minimum possible $\Delta Q_b$? [20 points]
Problem 4: (40 points)

Calculate the radius of the largest atom which may be located interstitially (without exercising stress) in a BCC lattice of a metallic system whose atoms have a diameter of $R$.

Problem 5: (40 points)

(a) What are the two conditions for Fermi-Dirac statistics that the fermions obey? [5x2 = 10 points]

(b) There is an assembly of $N$ non-interacting fermions under the conservation of particles and energy:

$$\sum_{j=1}^{n} N_j = N$$

$$\sum_{j=1}^{n} N_j \varepsilon_j = U$$

where $N_j$ is the number of particles with single-particle energy $\varepsilon_j$; $N$ and $U$ are fixed. There are $g_j$ quantum states at the $j$th energy level.

i. Derive the thermodynamic probability $\omega_j$ of this assembly of fermions for the $j$th energy level $\varepsilon_j$. [10 points]

ii. Using the above result, find the total number of microstates corresponding to an allowable configuration $\omega_{FD}$. [5 points]

iii. Express and Derive the Fermi-Dirac distribution function for a discrete energy level $\varepsilon_j$. *Hint:* Find the occupation number of each energy level when the thermodynamic probability is a maximum, i.e., the equilibrium macrostate $N_i$. Apply the method of *Lagrange multipliers* for a function with two variables under the conservation of particles and energy. Set one of the multipliers to be $1/kT$ and the other to be $\mu/kT$, where $\mu$ is the chemical potential. [3 + 12 points]