Modern/Optics/Thermodynamics  September 2007

Pick 4 out of 6 problems.

1  In inertial frame \( O \) a rod of length \( l \) is oriented along the \( x \)-axis and moving with velocity \( u \) in the positive \( y \) direction. This rod is then viewed from an inertial reference frame \( O' \) moving with velocity \( v \) in the positive \( x \) direction.

   a) What is the length of the rod in \( O' \)? [10 points]
   b) What angle does the rod make with respect to the \( x' \) axis? [30 points]

2  A cubic box (with sides of length \( L \)) holds diatomic \( \text{H}_2 \) gas at temperature \( T \). Each \( \text{H}_2 \) molecule consists of two hydrogen atoms with mass of \( m \) each separated by distance \( d \). Assume that the gas behaves like an ideal gas. Ignore the vibrational degree of freedom.
   a. What is the average velocity of the molecules? [10 points]
   b. What is the average velocity of rotation of the molecules around an axis which is the perpendicular bisector of the line joining the two atoms (assuming each atom as a point mass)? [10 points]
   c. Derive the expressions expected for the molar heat capacities \( C_p \) and \( C_v \) for such a gas. [10 x 2 points]

3.
   a. What is the threshold kinetic energy for the proton for \( p^+ \, n \rightarrow p^+ \, p^+ \, \pi^- \) assuming the neutron is at rest? [20 points]
   b. If now the neutron is in a Carbon nuclei of size 60 \( F^3 \) (with 6 protons and 6 neutrons), what is the Fermi energy of the neutron, and what is the threshold kinetic energy in this case? [20 points]

   \[ m_n = m_p = 1000 \, \text{MeV}, \quad m_\pi = 140 \, \text{MeV}, \quad \hbar c = 200 \, \text{MeV} \, \text{F} \]

4. A smooth vertical tube having two different sections is open from both ends and equipped with two pistons of different areas. Each piston slides within its respective tube section. One mole of ideal gas is enclosed between the pistons. The pistons are connected by a non-stretchable rod. The outside air pressure is 1 atm.

The total mass of the pistons is \( M \). The cross sectional area of the larger upper piston \( A_1 \), and the lower piston \( A_2 \) are related by \( A_1 = A_2 + \Delta A \).

How much (in Kelvin) must the inner gas (between the pistons) be heated in order to lift the piston assembly by \( L=5 \text{cm} \)?
5. Eight non-interacting neutrons are confined to a 3D square well of size $D = 5 \times 10^{-15}$ m such that $V = -50$ MeV for
\[0 < x < D,\]
\[0 < y < D,\]
\[0 < z < D\]
and $V = 0$ everywhere else.

a. How many energy levels are there in this well? [10 points]
b. What is the degeneracy of each energy level? [10 points]
c. What is the approximate Fermi energy for this system? [10 points]
d. What is the relative probability to be in the lowest energy state to the fourth lowest energy state at $kT = 10$ MeV? Just write down the ratio (don’t calculate the value). [10 points]

$m_n = 1000$ MeV, $\hbar c = 200 MeVF$

6. **Interference by a biprism.** A plane wave (wavelength $\lambda$) enters perpendicular to the biprism (the prism angle $\alpha$, refractive index $n$) as shown in the figure. The wave transmitted through both sides of the biprism is bent (refracted) and overlap at the viewing screen S (parallel to the biprism) where an interference pattern can be observed.

a. Find the refracted angle, $\beta$. [20 points]
b. Find the interval of adjacent interference lines. [20 points]