Problem 1. The half space $x > 0$ is filled with a constant magnetic field $\mathbf{B} = (0, 0, B_0)$, and the half space $x < 0$ is filled with a field in the opposite direction $\mathbf{B} = (0, 0, -B_0)$. An electron is shot out of the origin with initial velocity $\mathbf{v} = (-v_0/\sqrt{2}, -v_0/\sqrt{2}, 0)$. Describe its subsequent motion as quantitatively as possible.

Problem 2. A plane electromagnetic wave of angular frequency $\omega$ and peak electric field $E_0$ is moving in the $\hat{z}$ direction, and is polarized in the $\hat{x}$ direction. The wave is incident on a charge $Q$ of mass $m$. The charge is bound to the origin by a restoring potential

$$V(r) = m\omega_0^2 r^2 / 2,$$

where $r$ is the distance to the origin and $\omega_0$ is a constant. It is also acted on by a viscous damping force given by

$$\mathbf{F}_d = -m\omega_d \mathbf{v},$$

where $\mathbf{v}$ is the velocity of the charge, and $\omega_d$ is another constant.

(a) Find the position $\mathbf{r}$ of the charge as a function of time, ignoring magnetic effects. What is the phase angle between the electric field of the wave $\mathbf{E}(t)$ and $\mathbf{r}(t)$?

(b) Find the intensity of the scattered radiation as a function of direction and of $\omega$. (You do not need to evaluate the overall multiplicative factor.)
Problem 3. An azimuthally symmetric perfect conductor is approximately spherical, with mean radius \(R_0\), in the sense that the radius \(R\) as a function of the polar angle \(\theta\) is

\[ R(\theta) = [1 + \Delta(\theta)]R_0 \]

with \(\Delta(\theta) \ll 1\) and

\[ \int_{-1}^{1} \Delta(\theta) \, d(\cos \theta) = 0. \]

This conductor is given a total charge \(Q\). To first order in the function \(\Delta\), find the potential outside the conductor.

(You may leave your result in terms of the Legendre polynomials \(P_n(x)\), which satisfy:

\[ \int_{-1}^{1} P_n(x)P_{n'}(x) \, dx = \delta_{nn'} \frac{2}{2n + 1} \quad (1) \]

for non-negative integers \(n, n'\).)

Problem 4. The angular distribution of radiation emitted by an accelerated charged particle with charge \(q\) and velocity \(\vec{v} = c\vec{\beta}\) may be determined by

\[ \frac{dP(t')}{d\Omega} = \frac{q^2}{4\pi c} \left| \vec{n} \times [(\vec{n} - \vec{\beta}) \times (d\vec{\beta}/dt')] \right|^2 \]

where \(\vec{n}\) is the unit vector in the direction to the observer.

(a) What interpretation does \(t'\) have?

(b) What approximations, if any, are inherent to this expression?

(c) Suppose the charge undergoes linear acceleration, so that \(d\vec{\beta}/dt\) is parallel to \(\vec{\beta}\). Let \(\theta\) be the angle between \(\vec{\beta}\) and \(\vec{n}\). Find the locations \(\theta_{\text{max}}\) of the peak radiated intensity.

(d) How do these \(\theta_{\text{max}}\) values depend on the particle’s total energy \(E\) if the particle is highly relativistic?