Solve 3 out of 4 problems. (40 points each. Total of 120 points)

Do not just quote a result, show your work clearly step by step.

1. [40 points]
   (a) Find the force on a square loop placed as shown in Fig 1, near an infinite straight wire. Both the loop and the wire carry a steady current $I$. [20 points]
   (b) Suppose the current $I(t)$ in the long straight wire in Fig 1 is changing slowly according to $I(t) = I_0 \cos(\omega t)$. Find the current induced in the square loop as a function of time, if it has resistance $R$. [20 points]

2. [40 points] A capacitor is made of two plane parallel plates of width $a$ and length $b$ separated by a distance $d$ ($d \ll a$, $b$), as in Fig. 2. The capacitor has a dielectric slab of relative dielectric constant $K$ between the two plates.
   (a) The capacitance is connected to a battery of emf $V$. The dielectric slab is partially pulled out of the plates such that only a length $x$ remains between the plates. Calculate the force on the dielectric slab which tends to pull it back into the plates. [20 points]
   (b) With the dielectric slab fully inside, the capacitor plates are charged to a potential difference $V$ and the battery is disconnected. Again, the dielectric slab is pulled out such that only a length $x$ remains inside the plates. Calculate the force on the dielectric slab which tends to pull it back into the plates. [20 points]
3. [40 points] A soap film of thickness $a$ and permittivity $\varepsilon$ is suspended in empty space (Fig. 3). The permittivity of the soap film is very large compared to that of vacuum, $\varepsilon \gg \varepsilon_0$. Two charged ions, each of charge $+Q$, are located a distance $R$ apart, in the midplane of the film. Find the force between the ions in the three limiting cases:
   (a) if $R \ll a$ [13 points]
   (b) if $a \ll R \ll \varepsilon a$ [14 points]
   (c) if $\varepsilon a \ll R$ [13 points]

4. [40 points] Consider the following idealized situation with an infinitely long, thin, conducting wire along the $z$ axis. For $t < 0$ it is current-free, but at $t = 0$ a constant current density $\vec{J}$ is applied simultaneously over the entire length of the wire. Consequently, the wire carries the current
   \[
   \vec{J} = \begin{cases} 
   0, & t < 0; \\
   |\vec{J}| \hat{z}, & t \geq 0.
   \end{cases}
   \] (1)
   It is assumed that the conductor can be kept uncharged, i.e. $\rho = 0$.
   (a) Determine scalar and vector potentials induced everywhere in space, $\phi(\vec{x},t)$ and $\vec{A}(\vec{x},t)$ as functions of time. [10 points]
   (b) Determine the magnetic and electric fields induced everywhere in space, $\vec{B}(\vec{x},t)$ and $\vec{E}(\vec{x},t)$ as functions of time. [20 points]
   (c) Calculate the total power radiated per unit wire length. Comment on the unphysical behavior at $t = 0$ and its explanation for realistic systems. [10 points]