Problem 1: Electrostatic potential [40 points]

Consider an electrostatic potential $\phi(r) = \frac{Ae^{-\kappa r}}{r}$:

a) Using the Poisson equation, find the charge density distribution in the space except at the origin. (15 points)
b) The fact that the potential becomes infinity at the origin means the existence of a point charge at the origin. First obtain the electric field from the potential $\phi(r)$, then find the magnitude of the point charge and its sign. (15 points)
c) Find the total charge distributed other than the origin, also indicate its sign. (10 points)

Problem 2: A uniformly polarized ferroelectric slab is inserted into a parallel plate capacitor [40 points]

A uniformly polarized ferroelectric slab is inserted into a parallel plate capacitor as shown in the Figure. The capacitor area is $A$, and the lateral sizes are much larger than $a$, $b$. The two plates of the capacitor are connected through a thin wire.

a) Neglecting edge effects, determine the electric field everywhere in the capacitor. [16 points]
b) What is the electric field inside the ferroelectric plate for $b >> a$, and for vanishing $b$? [8 points]
c) Determine the force vector acting on the upper capacitor plate. [16 points]
**Problem 3:** A spherical shell with inner radius $a$ and outer radius $b$ with a uniform, fixed magnetization $M$ [40 points].

A spherical shell with inner radius $a$ and outer radius $b$ carries a uniform, fixed magnetization $M$ as shown in the figure. Using the method of separation of variables, find the vectors of the magnetic field and of the magnetic induction at:

a) $r < a$, [8 points]

b) $r > b$, [16 points]

c) $a < r < b$. [16 points]

**Hint:** a solution for the Laplace’s equation in spherical coordinates with azimuthal symmetry and z-axis along the magnetization direction.

Then the general solution (magnetic scalar potential) is

$$
\Phi_m(r, \theta) = \frac{U(r)}{r} P(\theta) = \sum_{l=0}^{\infty} \left[ A_l r^l + \frac{B_l}{r^{l+1}} \right] P_l(\cos(\theta)).
$$

**Problem 4:** Long concentric conducting tubes [40 points]

Consider a set of long concentric conducting tubes (length $l$) sharing the same axis along the tubes shown in Figure 3. The radii of the tubes are $a_1$ and $a_2$, respectively. At one end these tubes are connected via a resistor while the other ends are connected to a power supply, $\Delta \phi$.

(a) Find the magnetic field induced by the currents with the same magnitude but with the opposite directions.

(b) Find the self-inductance of this circuit. [10 points]