Problem EM1. In a vacuum diode, electrons with mass $m$ and charge $q$ are "boiled" off a hot cathode, at potential $V = 0$, and accelerated across a gap to the anode, which is held at positive potential $V = V_0$. The cloud of moving electrons within the gap (called space charge) quickly builds up to the point where it reduces the field at the surface of the cathode to zero. From then on, a steady current $I$ flows between the plates.

Suppose the plates are large relative to the separation (area $A \gg d^2$ in the figure below), so that edge effects can be neglected. Then the potential $V$, the charge density $\rho$, and the speed of the electrons $v$ are all functions of $x$ only.

(a) Assuming the electrons start from rest at the cathode, what is their speed at point $x$, where the potential is $V(x)$?

(b) In the steady state, the current $I$ is independent of $x$. What, then, is the relation between $\rho$ and $v$?

(c) Use the results above and Poisson's equation to obtain a differential equation for the potential $V(x)$, not involving $\rho$ or $v$, and solve it. (Hint: try a power law form.)

(d) Show that a non-linear relation holds between $I$ and $V_0$, of the form

$$I = KV_0^n$$

where $K$ is a constant that you will find in terms of the given quantities, and $n$ is a rational number.
Problem EM2. Consider the right circular cylindrical cavity pictured below. The inner and outer walls of the cavity, and the ends, are perfect conductors. The inner conductor has radius $a$, the outer conductor has radius $b$, and they are coaxial with the $z$-axis. The height of the cavity is $h$.

Consider the TEM modes of the cavity (the modes that have an electric field with no component in the $z$ direction), in the vacuum region $a \leq r \leq b$ and $0 \leq z \leq h$, with peak electric field $E_0$. Find expressions for the electromagnetic fields and the resonant frequencies and wavelengths of all such modes in terms of the given quantities.

![Diagram of a right circular cylindrical cavity](image)

Problem EM3. A long solenoid consists of $N$ turns of wire with resistance $R$ wound on a right circular cylinder of length $d$ and radius $a$, with $d \gg a$.

(a) Suppose that a slowly time-varying current $I(t) = I_0 \cos(\omega t)$ flows in the solenoid. Find the magnetic and electric fields everywhere inside and outside of the solenoid.

(b) A circular ring of wire with radius $b$ is coaxial with the solenoid and outside of it ($b > a$). Suppose now that a slowly varying current $I(t) = I_0 \cos(\omega t)$ flows in the ring. What current is induced in the solenoid?
Problem EM4. An electromagnetic wave is propagating in a non-conducting linear medium that has the same permeability as vacuum, $\mu = \mu_0$. The magnetic field of the wave is given by

$$\vec{B} = B_0 \left( \frac{\hat{y} + \hat{x}}{\sqrt{2}} \right) \cos \left( \frac{\sqrt{7} \omega}{2c} x + ay - \frac{\omega}{2c} z + \omega t \right)$$

where $a$ is a positive constant, and $c$ is the speed of light.

(a) Find $a$. What is the direction of propagation of the wave?

(b) What are the index of refraction and the permittivity of the material?

(c) Find the electric field $\vec{E}$ of the wave.

(d) The wave is incident on a square with side $d$ of some perfectly absorbing material that lies in the $y = 0$ plane. How much energy is absorbed by the material in time $T$? (Assume $T \gg 1/\omega$.)