NIU Ph.D. qualifier examination 2003 Fall (September 6, 2003)
Electricity and Magnetism
Solve 3 out of 5 problems.

I. Two infinite plane grounded conducting sheets intersect at an angle of \( \frac{\pi}{3} \). A charge \( q \) is placed equidistant from the conductors and at a distance \( a \) from their line of intersection. Sketch the E field in the region around charge \( q \), between its two adjacent conducting sheets. Find the force exerted on the charge by the conducting sheets.

II. A linear dielectric sphere of radius \( a \) and dielectric constant \( \kappa \) carries a uniform charge density \( \rho \), surrounded by vacuum.
   a) Find \( \mathbf{E} \) and \( \mathbf{D} \) inside and outside the sphere.
   b) Find the energy \( W \) of the system.

III. An infinite flat sheet of charge density \( \sigma \) per unit area, located in the xy-plane, is forced to oscillate along the x-axis. The velocity of charges at time \( t \) is given by \( \mathbf{v} = \mathbf{x} v_0 \cos(\omega t) \), resulting in electromagnetic radiation.
   (a) Solve for all components of the electromagnetic radiation.
   (b) How much energy per unit area is radiated away in a time \( T \)? (You may assume \( T >> \frac{1}{\omega} \))

IV. The electric field \( \mathbf{E} \) and the electric displacement \( \mathbf{D} \) in a certain linear anisotropic medium are related by an effective dielectric tensor \( \varepsilon_y \):

\[
D_j = \sum_{i=1}^{3} \varepsilon_{ij} E_i, \quad \varepsilon_{ij} = \varepsilon_0 \begin{pmatrix} 1 & ia & 0 \\ -ia & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]
where \( a \) is a positive constant less than 1. The indices 1, 2, 3 correspond to \( x, y, z \) respectively. The material is non-conducting and has the same magnetic permeability as vacuum \( (\mu = \mu_0) \).

(a) Given a plane wave propagating in the \( z \)-direction, find the polarization for which the medium has a definite index of refraction \( N \) \( (N = \frac{ck}{\omega}) \), and the corresponding values of \( N \).

(b) A semi-infinite slab of the above material fills the region \( z > 0 \). An electromagnetic plane wave of frequency \( \omega \) is incident normally on the flat direction of the material, and is therefore propagating in the \( z \)-direction. The wave enters the material from vacuum. The incident wave is linearly polarized with the electric field along the \( x \)-axis. Find the polarization of the reflected wave.

V. Suppose a photon of wavelength \( \lambda \) strikes a stationary electron and “bounce off” with a wavelength \( \lambda' \) at angle \( \theta \). This process is called Compton scattering; photons are neither created nor destroyed.

(a) Derive the Compton scattering formula relating \( \lambda', \lambda \) and \( \theta \).

(b) What part of the electromagnetic spectrum would best be used for experimental verification of Compton scattering, and why?