Problem 1  A uniform chain of length $L$ and total mass $M$ contains many links. It is held vertically by one end over a table with the other end just touching the table top.
(a) The chain is released and falls freely. What is the speed of the falling section of the chain at time $t$ after release? What is the force between the links? [10 points]
(b) Work out the increment of mass $dm$ that hits the table in an increment of time $dt$. Find the corresponding change in momentum and hence the instantaneous impulsive force on the table. [15 points]
(c) What is the total normal force acting on the table as a function of time? Show that the maximum value of the total force is $kMg$, where $k$ is a constant. Determine the value of $k$. [15 points]

Problem 2  Two identical thin uniform rods, each of length $d$ and mass $m$, are hinged together at the point $A$. The rod on the left has one end hinged at the fixed point $O$, while the end $B$ of the other rod slides freely along the horizontal $x$ axis. The system is in a uniform vertical gravitational field with acceleration $g$. All motion is frictionless.

(a) Find the total kinetic energy of the system. You should find the result:

$$T = md^2 \dot{\phi}^2 (a + b \sin \phi + c \sin^2 \phi)$$

where $a$, $b$ and $c$ are constant numbers that you will determine. (Exactly one of $a$, $b$, and $c$ is 0.) [25 points]
(b) Suppose the system is at rest at time $t = 0$ with $\phi = \phi_0$. What is the velocity of the hinge $A$ when it hits the horizontal $x$ axis? [15 points]
**Problem 3** A small spacecraft with mass $m$ and energy $E > 0$ approaches a star with mass $M$ and radius $R$ from far away.

(a) Find an expression for the effective total cross-section for the spacecraft to hit the star. [30 points]

(b) Sketch a graph of the cross-section as a function of $E$, and give simple physical explanations for the low and high energy limits. [10 points]

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**Problem 4** A wire in the shape of a parabola $z = \frac{r^2}{2a}$ rotates about the vertical $z$ axis with constant angular frequency $\Omega$. Here $r$ is the distance from the $z$ axis, and $a$ is a constant. A constant gravitational acceleration $g$ is directed in the negative $z$ direction. A small bead of mass $m$ slides on the wire without friction.

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(a) Find the Lagrangian and obtain Lagrange’s equations of motion for the bead, using $r$ as the coordinate. [14 points]

(b) There exists a solution of Lagrange’s equations for which $\dot{r} = \text{constant}$. What is this constant? From where does the energy come to permit such unbounded motion? [8 points]

(c) Obtain the equations of motion for small deviations of the bead from rest at the bottom of the parabola. Give a condition for stable oscillation about this position. [8 points]

(d) Find the canonical momentum, the Hamiltonian, and Hamilton’s equations of motion for the bead. Is the Hamiltonian conserved? [10 points]