Do ONLY THREE out of the four problems. Total points on each problem = 40.

**Problem 1.** Three rough cylinders of equal radius are stacked as shown on a rough level surface. They have unequal masses $M_1, M_2, M_3$, but the coefficient of static friction is everywhere equal to $\mu$. How large must $\mu$ be so that the arrangement is stable? Assume that cylinders 2 and 3 do not quite touch.

![Diagram of three cylinders](image)

**Problem 2.** Suppose that the gravitational interaction between two objects with masses $M$ and $m$ is modified by replacing the potential term in the Lagrangian according to:

$$\frac{GMm}{r} \rightarrow \frac{GMm}{r} \left(1 + \frac{\dot{r}^2}{C^2}\right)$$

where $r = |\vec{r}|$ with $\vec{r}$ the relative position vector, a dot means a time derivative, and $C$ = a new constant. In the following, assume that $m \ll M$ and treat the heavier object as stationary.

(a) Find the equations of motion for orbital motion in appropriate variables. [12 points]
(b) Does the radial vector of the small orbiting object sweep out equal areas in equal times? (Explain briefly.) [4 points]
(c) Find the angular frequency of small oscillations about a circular orbit of radius $R$. Write your answer in terms of only $G, M, m, R,$ and $C$. (Do not assume that $C$ is small.) [12 points]
(d) Let $M_E$ and $R_E$ be the mass and the radius of the Earth (assumed to be a non-rotating perfect sphere). Compute the escape velocity for an object initially on the surface of the Earth, as a function of $G, M_E, R_E, C$, and $\alpha$ = the angle between the initial velocity at the Earth’s surface and the vertical. [12 points]
Problem 3. A very heavy flat-bed truck starts at rest on a level surface. It has a ball which is a solid sphere of radius $R$ and mass $M$ (with uniform mass density) on the bed as shown, with its center a distance $d$ from the back end. When the truck starts moving, the truck accelerates uniformly with acceleration $a_T$, and the ball rolls without slipping on the truck bed. (Note: moment of inertia of a uniform sphere about the center is $I = \frac{2}{5}MR^2$.)

(a) How long does it take for the ball to roll off of the truck? [30 points]

(b) What density distribution in the ball (with same total mass $M$ and radius $R$) would minimize the time to fall off? What is this minimum time? [10 points]

Problem 4. Two beads, with unequal masses $m_1$ and $m_2$, are constrained to slide frictionlessly on a stationary hoop of radius $R$. The beads are connected as shown by two identical springs with spring constant $k$ and unstretched length $d$. There is no gravity.

(a) Find the normal modes of the system and their angular frequencies. [25 points]

(b) Now suppose that at time $t = 0$ the beads are on opposite sides of the hoop, with bead $m_1$ having an instantaneous speed $v$ and bead $m_2$ at rest. Solve for the subsequent motion of the first bead. [15 points]