1. Consider a projectile fired from the origin with a velocity \( \vec{v}_0 = \hat{i}v_{0x} + \hat{k}v_{0z} \) where \( z \) is the vertical coordinate. There is a wind with velocity \( \vec{v}_w = jv_w \) and air resistance is proportional to velocity \( \vec{F}_d = -b\vec{v} \).

(a) Find the position as a function of time for all three coordinates.
(b) Find the displacement in the \( x \)-position due to air resistance at \( z=0 \) keeping only the first order terms in \( b \).
(c) Find the displacement in the \( y \)-position due to the wind at \( z=0 \) keeping only the first order terms in \( b \).

2. Suppose at a latitude \( \lambda = 20^\circ\text{N} \) the atmospheric pressure is \( P = 10^5 \text{ N/m}^2 \) and the air density is \( \rho = 1.3 \text{ kg/m}^3 \).

(a) Determine an expression for the velocity \( v \) as a function of the pressure gradient and the radius \( r \).
(b) Use the result in part (a) to find the wind speed for a low pressure region with a pressure gradient of 3 millibar/m at 100 km from the center of low pressure.

3. Consider the motion of a particle of mass \( m \) moving on the outer surface of a hoop of radius \( R \). The particle is subject to the force of gravity on the Earth’s surface \( mg \). Use polar coordinates \( (r, \theta) \) as generalized coordinates to describe the motion of the particle.

(a) Find the Lagrangian for the particle in terms of the generalized coordinates.
(b) Use Lagrange’s equations to find an expression for the force of the constraint.
(c) Determine the angle at which the particle leaves the surface of the hoop.

4. A uniform string of length \( L \) and linear mass density \( \mu \) under tension \( T \) is displaced initially at rest as shown below.

(a) Write expressions for the initial conditions of the string.
(b) Find a general solution of the vibrating string as a Fourier series.
(c) Use the initial conditions to find the coefficients of the Fourier series.