

Instructions

This qualifying exam is scored based on the answers to 25 problems. Each problem is worth 4 points, and partial credit will be awarded for work done on a problem.

The problems are divided into six areas of physics. In each area there is a choice of problems to solve. If additional problems are attempted the best results will be selected for the score on the test.

Special Relativity (4 points): solve 1 of 2 problems.

Classical Mechanics (24 points): solve 6 of 8 problems.

Electromagnetism and Electronics (24 points): solve 6 of 8 problems.

Quantum Physics (24 points): solve 6 of 8 problems.

Waves and Optics (12 points): solve 3 of 4 problems.

Thermodynamics and Statistical Physics (12 points): solve 3 of 4 problems.

The exam will take four hours. Two hours will be given for the sections on Special Relativity, Classical Mechanics, and Electromagnetism and Electronics. Two hours will be given for the sections on Quantum Physics, Waves and Optics, and Thermodynamics and Statistical Physics.

You may use a calculator to find specific values required for some problems.

Useful constants:

$$g = 9.8 \text{ m/s}^2$$

$$c = 3.00 \times 10^8 \text{ m/s}$$

$$\epsilon_0 = 1 / c^2 \mu_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2$$

$$h = 6.63 \times 10^{-34} \text{ J s} = 1.24 \times 10^{-6} \text{ eV m/c}; hc = 1240 \text{ eV nm}$$

$$m_e = 0.511 \text{ MeV}/c^2, m_p = 938 \text{ MeV}/c^2$$

$$0^\circ\text{C} = 273.15 \text{ K}; N_A = 6.02 \times 10^{23}$$

Special Relativity: complete 1 problem

1. A particle has a momentum of $5 \text{ MeV}/c$ and relativistic energy of 10 MeV . Find the speed of the particle.
2. In the inertial frame of reference S , two events occur at the same instant in time and $3c \cdot \text{minutes}$ apart in space. In inertial frame S' , the same events occur $5c \cdot \text{minutes}$ apart. Find the time interval between the events in S' .

Classical Mechanics: complete 6 problems

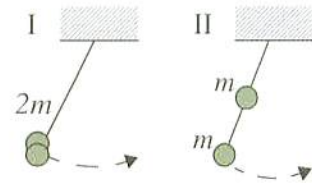
1. A coordinate transformation from a rotating polar coordinate system describing a particle in a plane to a fixed cartesian coordinate system is given by

$$x = r \cos(\theta + \omega t) \quad y = r \sin(\theta + \omega t)$$

Find the x -component of the velocity in the polar coordinate system.

2. A particle of mass m with an initial speed of 50.0 m/s moves in one dimension subject only to a resistive force $F = -cv^2$, where $c/m = 0.015 \text{ m}^{-1}$. Find the speed after $t = 2.0 \text{ s}$.
3. A thin uniform steel chain is 5.0 m long with a mass density of 1.6 kg/m. One end of the chain is attached to a horizontal axle having a radius that is small compared to the length of the chain. If the chain initially hangs vertically, find the work (in J) required to slowly wind it up onto the axle.

4. A long, straight, and massless rod pivots about one end in a vertical plane as shown in the figure at right. In configuration I, two small identical masses are attached to the free end. In configuration II, one mass is moved to the center of the rod. Find the ratio of the frequency of small oscillations of configuration II to that of configuration I.



5. A rigid cylinder of radius r rolls in a straight line at constant speed v without slipping on top of a horizontal plane surface. Find the magnitude and direction of the velocity and acceleration of the most forward point of the cylinder.
6. A gas with a uniform density ρ has a spherically symmetric flow pattern with velocity $-\frac{k}{r^2} \hat{r}$, where r is the distance from the origin. Find the amount of mass that flows per unit time from a region of volume V which includes the origin.

7. The total energy of a particle $E = T + U$, where T is the kinetic and U is the potential energy.

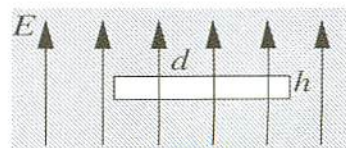
For a particle subject only to conservative forces, prove that $\frac{dE}{dt} = \frac{\partial U}{\partial t}$.

8. The force acting on a body of mass m constrained to move in a straight line is $-4kx^3$ where k is a constant. Find the Hamiltonian function for the system in terms of the position x and momentum p .

Electromagnetism and Electronics: complete 6 problems

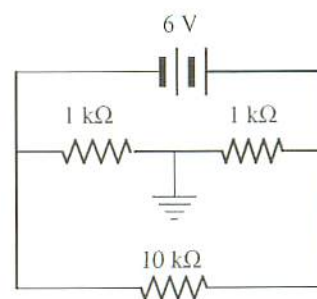
1. An electric field of magnitude E is uniform and parallel to the z axis. It passes through a hemisphere of radius R whose open end is in the xy -plane. Find the magnitude of the electric flux through the hemisphere.

2. A cylindrical cavity of diameter d and height h ($d \gg h$) is cut in a homogeneous, isotropic dielectric with a dielectric constant k . An electric field E is in the dielectric oriented perpendicular to the circular faces of the cavity as shown at right. Find the electric field within the cavity.



3. Two capacitors, $C_1 = 0.50 \mu\text{F}$ and $C_2 = 1.00 \mu\text{F}$, are connected in series with a 220 V DC power supply. Find the net charge on each capacitor plate.

4. Calculate the current supplied by the battery in the circuit at right.

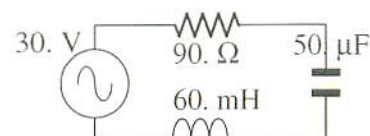


5. A point on the earth receives about 1300 W/m^2 of radiant energy from the overhead sun. Assume that this energy is in the form of a plane polarized monochromatic wave. Find the root-mean-square magnitude of the magnetic field of this wave in tesla.

6. A point on the earth has a magnetic field of $7.0 \times 10^{-5} \text{ T}$. This field is to be annulled by the magnetic field at the center of a circular conducting loop 5.0 cm in radius. Find the current required in the loop.

7. A sphere with a magnetic dipole of moment μ and a moment of inertia I about its center is placed in a uniform magnetic field B . Find the period of small oscillations of the dipole in the field.

8. In the circuit at right a 30. V ac source at 60 Hz is connected to a 90. Ω resistor, 50. μF capacitor, and 60. mH inductor in series. Find the tangent of the phase of the current through the circuit with respect to the voltage across the source.



Quantum Physics: complete 6 problems

1. A photon and an electron each have a wavelength of 0.5 nm. Find the ratio of the kinetic energy of the photon to the kinetic energy of the electron.
2. The wave function $\psi(x) = Ae^{-b^2x^2/2}$, where A and b are constants, is a normalized eigenfunction of the Schrodinger equation for a particle of mass M and energy E in a one dimensional potential $V(x)$ such that $V(x) = 0$ at $x = 0$. Find the value of the potential $V(x)$.
3. Consider two noninteracting electrons whose spin eigenfunctions are represented as follows: $\alpha(1)$ denotes the state with a total spin quantum number $S_1 = 1/2$, and a Z -component of the total spin quantum number $m_{s1} = 1/2$, and $\alpha(2)$ denotes a similar state for the second electron; $\beta(1)$ denotes the state with a total spin quantum number $S_1 = 1/2$, and a Z -component of the total spin quantum number $m_{s1} = -1/2$, and $\beta(2)$ denotes a similar state for the second electron. Find the total spin quantum number and Z -component of the total spin quantum number of a state represented by $(1/\sqrt{2})[\alpha(1)\beta(2) - \alpha(2)\beta(1)]$.
4. A photoelectric surface has a work function of 4.00 eV. Find the maximum velocity in m/s of the photoelectrons for incident light at a frequency of $3.00 \times 10^{15} \text{ s}^{-1}$.
5. An atom of radon-222 emits an alpha particle with a kinetic energy of 1 MeV. Estimate the de Broglie wavelength of the alpha particle in meters to the nearest order of magnitude.
6. Photons of wavelength λ scatter elastically on free protons initially at rest. Find the increase of the wavelength of photons scattered at 90° .
7. The absorption coefficient of aluminum (density 2.1 g/cm^3) is 0.73 cm^{-1} for x-rays with a wavelength 0.020 nm. Find the mass absorption coefficient.
8. In a star an alpha particle can fuse with a nucleus of ${}^7\text{N}^{14}$ to produce a new nucleus and a proton. Describe the composition of the new nucleus.

Waves and Optics: complete 3 problems

1. Small-amplitude standing waves of wavelength λ occur on a string of tension T , mass per unit length μ , and length L . One end of the string is fixed and the other end is attached to a ring of mass M that slides on a horizontal frictionless rod. Write an expression for μ/M in terms of the wavelength and length.
2. The screen of a pinhole camera is at a distance D from the pinhole, which has a diameter d . For light of wavelength λ , $\lambda \ll D$, determine the value of d which will give the sharpest image.
3. For blue light, a transparent material has a relative permittivity (dielectric constant) of 2.1 and a relative permeability of 1.0. Find the phase velocity of blue light in an unbounded medium of this material in units of the speed of light c .
4. A thin lens with refractive index $n_l = 1.50$ has a focal length f when the lens is in air. Find the focal length of the lens when it is immersed in water ($n_w = 1.33$).

Thermodynamics and Statistical Physics: complete 3 problems

1. A sample of 1.00 g of water with a volume of 1.00 cm^3 is at atmospheric pressure, $1.01 \times 10^5 \text{ N/m}^2$. The sample is kept at the same pressure and boiled, becoming 1671 cm^3 of steam. Given that the heat of vaporization of water is 539 cal/g at atmospheric pressure, find the increase in internal energy of the system.
2. A black body at a temperature of $227 \text{ }^\circ\text{C}$ radiates heat at the rate of $5 \text{ cal/cm}^2/\text{s}$. Find the heat radiated from that body at a temperature of $727 \text{ }^\circ\text{C}$.
3. An amount of ideal monatomic gas equal to 1 mole is at a constant pressure of 1 atm at a temperature of $0 \text{ }^\circ\text{C}$. Find the change of entropy of the gas as it is heated to $100 \text{ }^\circ\text{C}$.
4. A volume of oxygen gas, O_2 , is at 273 K and 1 atm pressure, and the molecules have a root-mean-square speed v . If the absolute temperature is doubled, and the oxygen is dissociated into atomic oxygen, find the root-mean-square speed of the oxygen atoms.