

Instructions - Part 1

This qualifying exam is scored based on the answers to 25 problems. Each problem is worth 4 points, and partial credit will be awarded for work done on a problem.

The problems are divided into six areas of physics. In each area there is a choice of problems to solve. If additional problems are attempted the best results will be selected for the score on the test.

Part 1 (2 hours)

Special Relativity (4 points): solve 1 of 2 problems.

Classical Mechanics (24 points): solve 6 of 8 problems.

Electromagnetism and Electronics (24 points): solve 6 of 8 problems.

Part 2 (2 hours)

Quantum Physics (24 points): solve 6 of 8 problems.

Waves and Optics (12 points): solve 3 of 4 problems.

Thermodynamics and Statistical Physics (12 points): solve 3 of 4 problems.

You may use a calculator to find specific values required for some problems.

Useful constants:

$$g = 9.8 \text{ m/s}^2$$

$$c = 3.00 \times 10^8 \text{ m/s}$$

$$h = 6.63 \times 10^{-34} \text{ J s} = 1.24 \times 10^{-6} \text{ eV m/c}; hc = 1240 \text{ eV nm}$$

$$m_e = 0.511 \text{ MeV}/c^2, m_p = 938 \text{ MeV}/c^2$$

$$N_A = 6.02 \times 10^{23}$$

Special Relativity: complete 1 problem

1. An atom moving at speed $0.3c$ emits an electron along the same direction with speed $0.6c$ in the internal rest frame of the atom. Find the speed of the electron in the lab frame.
2. A car of rest length 5 m passes through a garage of rest length 4 m. Due to Lorentz contraction, the car is only 3 m long in the garage's rest frame. Find the length of the garage in the car's rest frame.

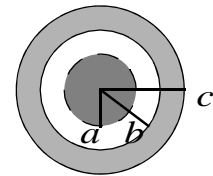
Classical Mechanics: complete 6 problems

- Let L be the Lagrangian of a system, and let the system be described by N generalized coordinates q_1, \dots, q_N . What constraint(s), if any, are required such that $\sum_{k=1}^N \dot{q}_k \left(\frac{\partial L}{\partial \dot{q}_k} \right) - L$ is a constant of the motion.
- Assume that a soap bubble of 2 cm diameter is formed on one end of a glass tube that has a closed valve at its center. A bubble of 4 cm diameter is formed at the other end of the tube from the same material as the first bubble. Describe what will happen to each bubble when the valve is opened and why.
- According to Kepler's third law, the period T of a planet is proportional to some power of its mean distance R from the sun. Write an expression for T in terms of R and identify any other needed variables or constants.
- A gas with uniform density ρ everywhere has a spherically symmetric flow pattern centered at a point where r is the distance from that origin. The velocity of each particle of the gas has a magnitude equal to k / r^2 directed toward the origin. Find the amount of gas that flows from a volume V which includes the origin.
- Let \vec{r} be the position vector, and let \vec{a} be a constant vector. Find the gradient of the scalar product $\vec{a} \cdot \vec{r}$.
- A rectangular coordinate system has axes with directions given by the unit vectors i, j , and k . The inertia tensor for a rigid body in that coordinate system is $I = K \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find the unit vector for the direction of the principal axis corresponding to the principal moment of inertia equal to K .
- Two point masses of 3.0 kg and 1.0 kg are attached to opposite ends of a horizontal spring with a spring constant of 300 N/m, and the system rests on a frictionless surface. Find the natural vibration frequency of this system in Hz.

8. Assume that six identical perfectly elastic balls are in a contiguous row on a straight frictionless track. The row of balls is struck by an elastic ball of the same diameter but twice the mass of the other individual balls. Find the number of balls which recoil as a result of this collision.

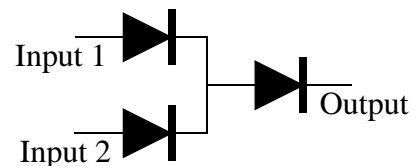
Electromagnetism and Electronics: complete 6 problems

1. A point charge q is placed a distance a from a grounded infinite plane conducting surface. Find the charge induced on the surface.
2. A sphere with a magnetic dipole of moment μ and moment of inertia I about its center is placed in a uniform magnetic field B . Find the period of small oscillations of the dipole.
3. A conducting sphere of radius a is surrounded by a dielectric shell of inner radius b and outer radius c with dielectric constant k as shown at right. A total charge Q is placed on the conducting sphere. Assume that the zero of potential is at infinity, and find the potential of the conducting surface.



4. In a half-wave power supply there is a component often called the filter capacitor. Briefly describe whether the filter capacitor should be before or after the rectifier and describe the purpose of the filter capacitor.
5. An ion of mass M and charge Q is circulating in a uniform magnetic field B with a radius of curvature R . Find the equivalent voltage V to give the ion the same acceleration (non-relativistic).

6. In the circuit at the right the components are ideal. The values in the circuit are 0 V for logical 0 and +1 V for logical 1. Make a truth table for the circuit and identify the common name for the operation.



7. A solenoid has an inductance of 0.32 henry. Find its reactance to an alternating current having a frequency of 1.0 kHz.
8. An electrometer sensitive to small amounts of charge has a capacitance of 2 pF and a sensitivity of 2×10^{-2} V/cm deflection. Charges are collected in an ionization chamber with a capacitance of 5 pF. Find the order of magnitude of the number of electrons needed to cause a deflection of 0.1 cm on the electrometer.

Instructions - Part 2

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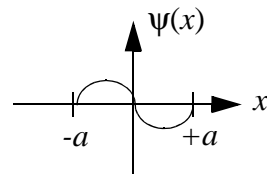
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Quantum Physics: complete 6 problems

1. The symbols used to represent the first four values of the orbital quantum number for atomic electrons are $s p d f$. List all possible values of the total angular momentum for a single f electron.
2. The wave function of a particle is $e^{i(kx-\omega t)}$ where x is distance, t is time and k and ω are positive real numbers. Find the x -component of the momentum of the particle.

- A hydrogen atom with Bohr radius a_0 is in its ground state in the presence of a static electric field E . Write an expression for the first order correction to the energy of the hydrogen based on perturbation theory.
- Photoelectrons emerge from a surface due to light at 400 nm. Find the change in energy in eV of the photoelectrons for light at 300 nm (don't forget the sign of the change).

- The figure at right shows one of the possible energy eigenfunctions $\psi(x)$ for a particle bouncing freely back and forth between impenetrable walls located at $x = -a$ and $x = +a$. The potential energy equals zero for $|x| < a$. If the energy of the particle is 2 eV when it is in the quantum state associated with this eigenfunction, find the energy when it is in quantum state of lowest possible energy.



- In the near infrared spectrum of carbon monoxide (CO) there is an intense band at 2144 cm^{-1} . Find the fundamental vibration frequency of CO in Hz.
- The order in which electrons fill the outer atomic shells of atoms deviates from a uniform increase in the quantum number n ; for example, instead of filling the $3d$ level first the $4s$ level is filled first. Briefly describe the reason for this including any particular interactions involved.
- A Ba^{137} nucleus in an excited state emits a photon of 0.66 MeV. Estimate the recoil kinetic energy of the nucleus in eV.

Waves and Optics: complete 3 problems

- A whistle whose air column is open at both ends has a fundamental frequency of 5000 Hz. Find the length of the whistle to the nearest cm.
- A telescope has an aperture of 1.27 m diameter. Find the smallest angular separation in radians that can be resolved by the telescope for light with a wavelength of 540 nm.
- A parallel beam of light of energy density U falls normally on an object and is totally reflected. Find the pressure exerted by the light on the object.
- A ray of light with wavelength λ strikes at normal incidence on a thin transparent film with an index of refraction n . Find the minimum thickness to have constructive reinforcement of the reflected ray.

Thermodynamics and Statistical Physics: complete 3 problems

1. A classical model of a diatomic ideal gas is two balls connected by a spring, and the system is free to rotate about axes perpendicular to the spring. Using this model find the specific heat per mole at constant pressure of this diatomic gas at high temperature in terms of the gas constant R .
2. A glowing incandescent lamp filament may be regarded as a black body. If the filament is heated to 2400 K with a power of 100 W, find the surface area of the filament in cm^2 .
3. Let T_1 and T_2 be the temperatures of two reservoirs where $T_1 > T_2$. Find the efficiency of a Carnot engine operated between these two reservoirs.
4. Draw a P-V diagram for an monatomic gas undergoing an adiabatic expansion. Indicate the direction of the change of the system on the graph and shade the appropriate area to indicate the work done by the gas. Draw a dashed line on the diagram to show an isotherm.