

Name:

MATH 230
MOCK FINAL EXAM

Disclaimer: This mock exam is for practice purposes only. No graphing calculators \geq TI-89 is allowed on this test. Be sure that all of your work is shown and that it is well organized and legible.

Good luck!

Calculus Tutoring Center
DU 326

1. (16 points)

(a) Sketch the region bounded by $y = e^x$, $y = e$, and $x = 0$.

(b) Set up, but do not evaluate, an integral which determines the volume of the solid generated by revolving the region in (a) about the x -axis.

(c) Set up, but do not evaluate, an integral which determines the volume of the solid generated by revolving the region in (a) about the y -axis.

2. (16 points) Compute the derivatives of the following functions. Do not simplify your answer.

(a) $f(x) = \tan^{-1}(1 - 2^x)$

(b) $f(x) = (x^2 + 2)^x$

3. (10 points) Find the length of the curve $y = \frac{x^2}{8} - \ln x$ from $x = 1$ to $x = 4$.

4. (12 points) Let $f(x) = 2e^{3x} + x$

(a) Use $f'(x)$ to explain why $f^{-1}(x)$ exists on $(-\infty, \infty)$.

(b) Find $(f^{-1})'(2)$. Note: $f(0) = 2$.

5. (16 points) Compute the following limits

(a) $\lim_{x \rightarrow 0} \frac{x^2}{1 - \cos(2x)}$

(b) $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{2n}\right)^{3n}$

6. (50 points) Evaluate the following integrals:

(a) $\int \frac{3x - 5}{x^2 - 4x + 3} dx$

(b) $\int \sec^4(x) dx$

(c) $\int \frac{x^2}{\sqrt{4 - x^2}} dx$

(d) $\int x^2 \ln x \, dx$

(e) $\int_{1/2}^{\infty} \frac{4}{1+4x^2} \, dx$

7. (10 points) Find the sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{(-1)^n}{3^n} \right)$

8. (16 points) For each of the following series, determine whether it is convergent or divergent. Justify your answer.

(a)
$$\sum_{n=2}^{\infty} \frac{1}{(n+1) \ln(n+1)}$$

(b)
$$\sum_{n=1}^{\infty} \frac{1}{n+2^n}$$

9. (12 points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. Justify your answer.

(a)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin(n)}{n(n+1)}$$

(b)
$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{n+1}{n^2 \ln(n)}$$

10. (12 points) Find the radius of convergence and interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\frac{x-3}{2} \right)^n$.

11. (8 points) Find the Taylor Series centered at $a = 0$ for $\int \cos(x^2) dx$ by using the Taylor Series for $\cos(x)$ at $a = 0$.

12. (10 points)

(a) Determine the third degree Taylor Polynomial $P_3(x)$ for $f(x) = \ln(2 + x)$ at $a = -1$.

(b) Is $P_3'(x)$ the second degree Taylor Polynomial for $f'(x)$ at $a = -1$? Explain.

13. (6 points) Is $\sum_{n=1}^{\infty} a_n$ a series for which $\sum a_n = -2$, and s_n is the n -th partial sum for the series, determine:

(a) $\lim_{n \rightarrow \infty} a_n =$

(b) $\lim_{n \rightarrow \infty} s_n =$

14. (6 points) If $\sum c_n x^n$ is a convergent power series for $x = 3$ but not $x = 5$, can you conclude that it converges for (i) $x = -2$, (ii) $x = -3$, (iii) $x = -6$. Explain.