Exam 3 Study Guide

1. Find the maximum and minimum values of the function $f(x) = x^3 - 3x + 1$ over the interval $[-3, 0]$.

2. Find the maximum and minimum values of the function $f(x) = \sin x + \cos x$ over the interval $[0, 2\pi]$.

3. State the Mean Value Theorem.

4. If $f'(x) \leq 5$ for all x and $f(1) = -2$, use the Mean Value Theorem to show why $f(5) \leq 18$.

5. Find the intervals of increase/decrease and concavity, all local maxima/minima, and inflection points, for the function $f(x) = x^3 - 6x^2 - 15x + 4$.

6. Find the intervals of increase/decrease and concavity, all local maxima/minima, and inflection points, for the function $y = \sin^2 x - 2\cos x$.

7. Use Rolle's Theorem to explain why the equation $2x - 1 - \sin x = 0$ has at most one root.

8. Sketch a function with domain $[0, \infty]$ with the following traits:

$f(0) = 0$ $\qquad$ $f'(0) < 0$ $\qquad$ $f(4) = 2$

$\lim\limits_{x \to 10} = \infty$ $\qquad$ $x = 5$ is an maximum

$\lim\limits_{x \to \infty} = 4$ $\qquad$ $x = 6$ is an inflection point

9. Evaluate the following infinite limits, demonstrating all necessary work.

(a) $\lim\limits_{x \to \infty} \dfrac{1 + x^2 - 2x^4}{6x^4 - 3x^2 - 4}$

(b) $\lim\limits_{x \to \infty} \dfrac{x^3 + 3}{\sqrt{4x^6 - 3x^2 + x}}$

10. Evaluate the following infinite limits, demonstrating all necessary work.

(c) $\lim_{x \to \infty} (\sqrt{x^2 - 1} - x)$

(d) $\lim_{x \to \infty} \sqrt{x} \sin \frac{1}{\sqrt{x}}$

11. Using $x_1 = 1$, find the next two approximations of the root of $f(x) = x^3 + x - 1$ using Newton's method.

12. Demonstrate how Newton's method can be used to find an approximation of $\sqrt[3]{2}$, using $x_1 = 1$ to calculate x_2.

13. Find the minimum distance from the point $(1, 1)$ to the line $y = 2x - 2$.

14. Find the maximum area of a rectangle whose base is the x-axis and whose upper corners rest on the parabola given by $y = 4 - x^2$. (note that the upper corners must have $y > 0$ for their points).

15. A farmer fences in a rectangular pigsty adjacent to his barn (so that he doesn't have to fence the side running along the barn). If the farmer wants to enclose 800 square feet and fence costs $50 per foot, what is the minimum cost of the fencing needed?

Given the following derivatives, find the original functions.

(a) $f'(x) = 3x^2 + 4x^3 + \frac{2}{x^4}$

(b) $g'(x) = \dfrac{x^3 - 4x^2 + 7}{x^2}$

(c) $h'(x) = \dfrac{1 + \sin x}{\cos^2 x}$

(d) $f'(x) = 5x - 20,\ f(1) = 2$

(e) $g''(x) = x - \cos x, g(0) = 4, g'(0) = 2$

16. Approximate the area under the curve $y = 9 - x^2$ over the interval $[0, 2]$, using four approximating rectangles. Show both left and right endpoint approximations.

17. Find a region whose area is equal to the limit $\lim_{n\to\infty} \sum_{i=1} n \frac{3}{n} \left(1 + \frac{3i}{n}\right)^3$

Section 4.5 problems

18. Find the following indefinite integrals by using u-substitution

(a) $\int x\cos(x^2)dx$

(b) $\int \sin 3x dx$

(c) $\int 2x^5\sqrt{x^2-1}dx$

(d) $\int \tan x \sec^2 x dx$

(e) $\int -\frac{\cos\left(\frac{1}{x}\right)}{x^2}dx$