Determine where \( f(x) \) is continuous.

\[
f(x) = \begin{cases} 
(x + 2)^2 - 1, & x < -1 \\
2, & x = -1 \\
-2x + 3, & -1 < x < 1 \\
\sqrt{x}, & x \geq 1 
\end{cases}
\]

1. There are four pieces to this function: \((x + 2)^2 - 1\), \(2\), \(-2x + 3\), and \(\sqrt{x}\). The three functions \((x + 2)^2 - 1\), \(-2x + 3\), and \(\sqrt{x}\) are continuous on their respected domain (except possibly at their endpoints). This means

    (a) \((x + 2)^2 - 1\) is continuous on \((-\infty, -1)\).

    (b) \(-2x + 3\) is continuous on \((-1, 1)\).

    (c) \(\sqrt{x}\) is continuous on \((1, \infty)\)

2. The only places we are concerned now is \(x = -1\) and \(x = 1\). Let’s check to see if they satisfy the three conditions of continuity.

3. Is \(f(x)\) continuous at \(x = -1\)?

    (a) Does \(f(-1)\) exist?

        Yes. The second part of the function states \(f(-1) = 2\).

    (b) Does \(\lim_{x \to -1} f(x)\) exist?

        No. But we need to prove it. We need to check the left and right hand endpoints.

        \[
        \lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (x + 2)^2 - 1 = (-1 + 2)^2 - 1 = 0
        \]

        \[
        \lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} -2x + 3 = -2(-1) + 3 = 5
        \]

        Since \(\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)\), the general limit does not exist.

        Therefore, \(f(x)\) is not continuous at \(x = -1\).
4. Is $f(x)$ continuous at $x = 1$?

(a) Does $f(1)$ exist?

Yes. $f(1) = \sqrt{1} = 1$

(b) Does $\lim_{x \to 1} f(x)$ exist?

Yes. But we need to prove it. We need to check the left and right hand endpoints.

\[
\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} -2x + 3 = -2(1) + 3 = 1
\]

\[
\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \sqrt{x} = \sqrt{1} = 1
\]

Since $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$, the general limit exists, $\lim_{x \to 1} f(x) = 1$

(c) Does $\lim_{x \to 1} f(x) = f(1)$?

Yes. You can see (a) and (b) match.

Therefore, $f(x)$ is continuous at $x = 1$.

5. Final Answer: $f(x)$ is continuous everywhere except at $x = -1$. We can also write that as

$$(-\infty, -1) \cup (-1, \infty)$$