Use the chain rule to find the derivative, \( f'(x) \).

Recall, for the function \( h(x) = f(g(x)) \), the chain rule is

\[
h'(x) = f'(g(x)) \cdot g'(x)
\]

1. \( h(x) = \sqrt[3]{1 + \tan(x)} \)

   (a) Make sure you rewrite the function in a form you can differentiate. Instead of \( \sqrt[3]{u} \), we use \((u)^{1/3}\)

   \[
h(x) = (1 + \tan(x))^{1/3}
\]

   (b) Let’s get started

   \[
h'(x) = \frac{1}{3} (1 + \tan(x))^{-2/3} \cdot \frac{d}{dx} [1 + \tan(x)]
   = \frac{1}{3} (1 + \tan(x))^{-2/3} \cdot (0 + \sec^2(x))
   = \frac{1}{3} (1 + \tan(x))^{-2/3} \cdot (\sec^2(x))
\]

2. \( h(x) = (7x^3 - 14x + 1)^{125} \)

   \[
h'(x) = 125 (7x^3 - 14x + 1)^{124} \cdot \frac{d}{dx} [7x^3 - 14x + 1]
   = 125 (7x^3 - 14x + 1)^{124} \cdot (21x^2 - 14)
\]

3. \( h(x) = \sin(x^2 \cdot \cos(x)) \)

   This is a multiple rule problem. It will require not only the chain rule, but the product rule as well. If you don’t see it yet, you will in a moment. Let’s get started.

   \[
h'(x) = \cos(x^2 \cdot \cos(x)) \cdot \frac{d}{dx} [x^2 \cdot \cos(x)]
   x^2 \cdot \cos(x) \text{ requires the product rule}
\]
Let’s do the product rule on $x^2 \cos(x)$

\[ f(x) = x^2 \quad g(x) = \cos(x) \]
\[ f'(x) = 2x \quad g'(x) = -\sin(x) \]

\[ \frac{d}{dx} \left[ x^2 \cdot \cos(x) \right] = -x^2 \sin(x) + 2x \cos(x) \]

Now let’s finish the problem.

\[ h'(x) = \cos(x^2 \cdot \cos(x)) \cdot \frac{d}{dx} \left[ x^2 \cdot \cos(x) \right] \]
\[ = \cos(x^2 \cdot \cos(x)) \cdot (-x^2 \sin(x) + 2x \cos(x)) \]