1. (20 points) Determine the area of the bounded region given by the curves \( y = 3x^2 - 15 \) and \( y = 18x + 6 \).
2. (15 points) The integral $\int_0^1 \sqrt{1 + e^{2x}} \, dx$ represents the arc-length along the graph of $e^x$ from the point $(0, 1)$ to the point $(1, e)$.

(a) Use the mid-point rule with $n = 2$ to approximate the integral.

(b) The straight-line segment from $(0, 1)$ to $(1, e)$ has length $\sqrt{1 + (e - 1)^2}$. Is the approximation greater or smaller than this value? Are the values reasonably close?
3. (15 points) Half of a solid spherical ball of gold of radius 1 rests on a table with the flat side down. It is cut into two pieces along a plane parallel to the table at height $\frac{1}{3}$. One of the two pieces has two flat sides, the other has one flat side. Which is the heavier of the two pieces?
4. (10 points) Setup and simplify as far as possible, but do not evaluate, the integral that gives the length of the portion of the ellipse
\[ \frac{x^2}{4} + y^2 = 1 \]
in the first quadrant from (0, 1) to (2, 0).

5. (10 points) Determine the simplest form of the formula for the surface area of a cone when the height of the cone is assumed equal to \( h \) and the radius of its opening is \( r \).

Hint: Revolve the straight line \( y = mx \) about the \( x \)-axis to generate the cone.
6. (15 points) Determine if the following limits exist and if so calculate their value.

(a) \( \lim_{x \to 1} \frac{5x^4 - 4x^2 - 1}{10 - x - 9x^3} \)

(b) \( \lim_{x \to 0^+} x \ln x \)

(c) \( \lim_{x \to \infty} (\sqrt{x^2 + x} - x) \)
7. \((30 \text{ points})\) Calculate the following integrals.

(a) \(\int x \sin(x) \, dx\)

(b) \(\int \cos^3(x) \, dx\)

(c) \(\int \frac{4x^4}{x^2 - x^2 + x - 1} \, dx = \int \frac{4x^4}{(x-1)(x^2+1)} \, dx\)
8. (10 points) Calculate the following integrals.

(a) \[ \int \frac{x}{\sqrt{1-x^2}} \, dx \]

(b) \[ \int \frac{x^2}{\sqrt{1-x^2}} \, dx \]
9. (10 points) Determine if the following integrals converge, and if so calculate the value.

(a) \( \int_{-1/5}^{4} \frac{2}{x^2} \, dx \)

(b) \( \int_{0}^{\infty} x \, e^{-x^2} \, dx \)
10. (10 points) Let $f(x) = \tan(x)$.

(a) Determine the third degree Taylor polynomial $P_3(x)$ of $f(x)$ with $a = 0$.

(b) What is the actual error $P_3\left(\frac{\pi}{4}\right) - f\left(\frac{\pi}{4}\right)$ when $P_3$ is used in place of $f$?
11. (15 points) Let \( f(x) = \frac{1}{2 + 3x} \).

(a) Determine the power series representation of \( f(x) \) in the form 
\[ \sum_{n=0}^{\infty} a_n x^n. \]

(b) Determine the interval of convergence of the power series and the radius of convergence.
12. (15 points) Consider the power series \( f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \)

(a) Determine the interval of convergence of \( f(x) \).

(b) Use the power series \( \frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \) to derive the power series of \( g(x) = \frac{1}{1+x^2} \).

(c) Which mathematical operation turns the power series of \( g(x) \) into \( f(x) \)?

(d) Which function \( h(x) \) is produced when this mathematical operation is applied to \( g(x) \)?

(e) What is the domain of \( h(x) \), and is it different from the domain of \( f(x) \)?
13. (25 points) Answer the following questions. Indicate clearly any test you employ.

(a) Determine if \( \sum_{n=0}^{\infty} (-1)^n \cdot \frac{e^n}{3^n} \) converges or diverges. If it converges, then calculate its value.

(b) Determine if \( \sum_{n=1}^{\infty} \frac{\sqrt{2n^3-1}}{n^3+5} \) converges or diverges.

(c) Determine if \( \sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right) \) converges or diverges.

(d) Determine if \( \sum_{n=1}^{\infty} (-1)^{n+1} (e^{1/n} - 1) \) converges or diverges.

(e) Determine if \( \sum_{n=1}^{\infty} \frac{2^n n!}{n^n} \) converges or diverges.