JOURNAL OF THE AMERICAN WATER RESOURCES ASSOCIATION
AUGUST AMERICAN WATER RESOURCES ASSOCIATION 2003

A THEORETICAL TRAVEL TIME BASED ON WATERSHED HYPSOMETRY[1]

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ABSTRACT: The time to hydrograph peak of a watershed basin has been found to correlate with various statistical attributes (e.g., skewness and kurtosis) of its hypsometric curve (treated as probability distribution). This paper presents a theoretical travel time that is conceptually analogous to the time to hydrograph peak and can be calculated directly from the hypsometric curve of a watershed basin based on gravity and acceleration. The theoretical travel times for 23 selected watersheds in the United States are found to correlate significantly with their corresponding hypsometric attributes. In addition, the theoretical travel times are consistent with the times of concentration estimated from the Federal Aviation Administration method. Thus, this paper offers a simple theoretical explanation to the empirically identified linkage between time to hydrograph peak and hypsometric attributes. This theoretical travel time can provide an alternative way of characterizing the effects of basin morphometry on hydrologic response.
(KEY TERMS: surface water hydrology; Geographic Information Systems; travel time; hypsometry; DEM.)

Luo, Wei and John M. Harlin, 2003. A Theoretical Travel Time Based on Watershed Hypsometry. J. of the American Water Resources Association (JAWRA) 39(4):785-792.

INTRODUCTION

It has long been recognized that hydrologic response of a watershed basin and its geomorphologic characteristics are interrelated. Many geomorphic parameters have been identified by empirical correlation and are used in models to predict the hydrologic response. Parameters often used include those characterizing basin size, shape, relief, slope, and its drainage network (e.g., Black, 1972; Paton and Baker, 1976; Beven and Kirkby, 1979; Rodriguez-Iturbe and Valdes, 1979; Gupta *et al.,* 1980). However, the empirical approach produced largely inconclusive and often conflicting or region specific results regarding the role of basin morphometry and relief on basin hydrologic response (Howard, 1990). Of the parameters often used, only basin area and some measure of basin relief are found to correlate more or less consistently at a significant level with flood response (peak discharges or time to hydrograph peak) (Howard, 1990). Basin area and relief have been elegantly combined in hypsometric curve (Strahler, 1952). Thus hypsometric curve should be explored to see if it can offer some better ways of characterizing the effect of a basin's morphometry on its hydrologic response.

The hypsometric curve, which describes the proportion of a basin's area that is above a certain elevation (Figure 1), represents an overall basin slope and embodies much of the geomorphic information of a watershed (Strahler, 1952). It has been demonstrated that by treating the hypsometric curve as a cumulative probability distribution function, the form of the curve can be quantitatively described by a series of statistical moments, such as skewness and kurtosis of the curve and skewness and kurtosis of its density function (see Harlin, 1978, for detail and also refer to the Appendix of this paper for definition of these parameters). Further, these parameters or attributes are sensitive to subtle changes in overall basin slope and basin development as the mass is removed by erosion over a long geological time period (Harlin, 1980). For example, headward development of the main stream and its tributaries produces increasing values for hypsometric skewness (Harlin, 1980). In addition, the density function of the curve is related to rates of change in overall basin slope and tendency

[1]Paper No. 01236 of the *Journal of the American Water Resources Association.* **Discussions are open until February 1, 2004.**
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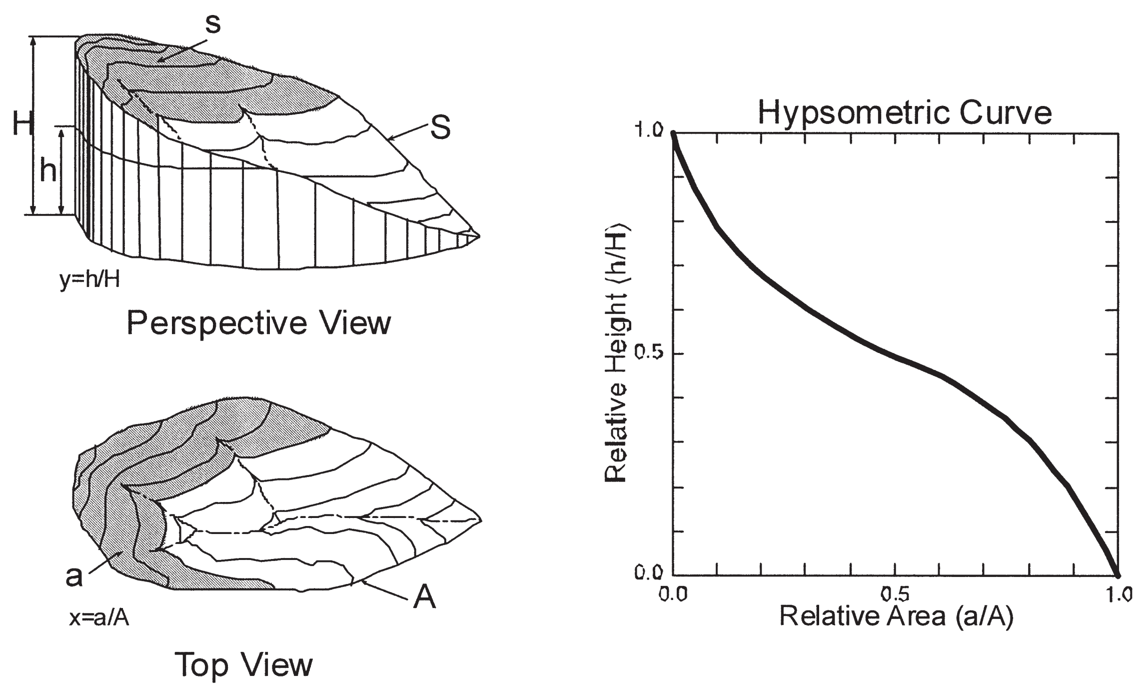

Figure 1. Schematic Diagram Illustrating the Hypsometric Curve and the Variables Involved. The curve essentially describes the distribution of area with elevation (i.e., the relative proportion of a region's area that lies at or above a given height relative to the total elevation range in the area under consideration) (a = area of basin above height h; h = height above outlet; A = total area of basin; and H = total relief of basin). Figure adapted after Strahler (1952).

toward geomorphic equilibrium (Harlin, 1980). The notion that moments and centers of gravity of the hypsometric curve were sensitive to rates of change in watershed slope, as well as geomorphic development, suggested that hypsometric curve could be used to more accurately describe potential energy distribution throughout a watershed and to distinguish the difference in potential energy between drainage basins (Harlin, 1978, 1980).

Harlin (1984) successfully established an empirical correlation between the hypsometric parameters of a watershed basin and its observed time to hydrograph peak (defined as the time difference between the centroid of the hyetograph and peak of the hydrograph) but did not offer a direct theoretical explanation for that relationship. The purposes of this paper are to: (1) develop a theoretical travel time directly based on hypsometric curve and physical laws that can be used as an alternative means of characterizing the effect of basin morphometry on hydrologic response; and (2) provide a theoretical basis for the previously identified empirical relationship between time to hydrograph peak and basin hypsometric attributes.

THEORETICAL TRAVEL TIME DERIVED FROM HYPSOMETRIC CURVE

The time to hydrograph peak is defined in Harlin (1984) as the time difference from the centroid of an intense rainfall hyetograph to the associated peak in the hydrograph. It is also called lag to peak or lag time (e.g., Singh, 1992; Dingman, 1994). As indicated in previous studies, the lag time is influenced by many factors, e.g., basin morphometry, soil characteristics, vegetation cover, rainfall intensity, land use, preexisting wetness conditions, and so on. To focus on the effect of morphometry on basin hydrologic response, a conceptually simple approach is taken in this paper to determine the travel time of a water drop moving down an impermeable and frictionless surface represented by the basin's hypsometric curve (Figure 2). This travel time depends only on the physical laws (i.e., the tangential component of gravity and associated acceleration) and the overall slope of the watershed (represented by its hypsometric curve). In essence, this approach allows for comparison of the effect of basin morphometry on hydrologic response among different basins, assuming all other factors are the same. In other words, if all watersheds have the same smooth impervious surface (e.g., concrete), what makes their hydrologic response different is their morphometry. This is not to say that other factors are not important. They are simply not the focus of the paper. The advantage of this approach is that it simplifies the derivation considerably by working with a two-dimensional overall slope profile instead of the full three-dimensional basin. Further, unlike many hydrologic models, it takes the overall basin slope rather than the local slope into consideration and it is directly linked to the potential energy distribution throughout the basin.

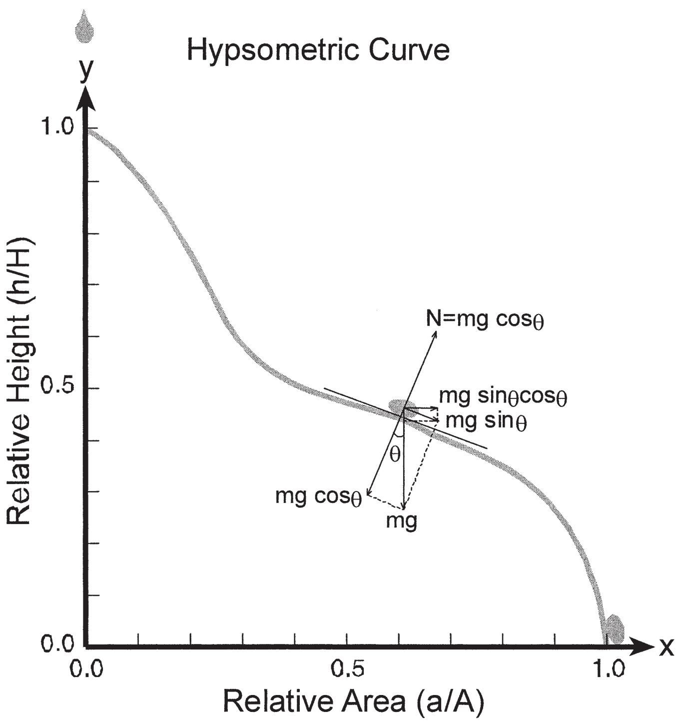

Figure 2. The Force Analysis of a Drop of Water Moving Down a Frictionless Surface Represented by the Hypsometric Curve of a Watershed Basin (g = gravitational acceleration constant; a = acceleration in the direction parallel to the tangent of the slope at $x = x$; m = mass of the drop of water; and θ = slope angle at $x = x$).

The hypsometric curve can be assumed to take the following polynomial function form (Harlin, 1978).

$$y = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n \tag{1}$$

where y = relative height, x = relative area, and c_i = polynomial coefficient, $0 \le i \le n$. Practically, $n = 3$ would be sufficient, because for most watersheds $n = 3$ would result in $r^2 > .9$ at a significant level of 0.0001 (Harlin, 1978).

The slope of the curve at any point $x = x$ can be expressed as the first derivative of the polynomial function.

$$\tan\theta = y' = c_1 + 2c_2x + \cdots + nc_nx^{n-1} \tag{2}$$

where θ is the slope angle (i.e., the acute angle between a horizontal line and the tangential line at any point $x = x$ on the curve).

For a drop of water with mass m, the balance of force in the direction parallel to the tangent of the frictionless slope at $x = x$ is (see Figure 2)

$$mg \sin\theta = ma \tag{3}$$

where g = gravitational acceleration constant and a = acceleration in the direction parallel to the tangent of the slope at $x = x$.

The horizontal component of the acceleration is

$$a_x = a\cos\theta = g\sin\theta\cos\theta = g\frac{\tan\theta}{1+\tan^2\theta} \tag{4}$$

At time t, the horizontal component of the velocity is $v_x(t)$; from t to $t + dt$, the drop traveled dx, and the velocity change is $dv_x(t)$, then

$$dx = v_x dt \tag{5}$$

$$dv_x = a_x dt \tag{6}$$

where v_x is the horizontal velocity at $x = x$.

From Equation (5)

$$dt = \frac{dx}{v_x} \tag{7}$$

Substitute Equation (7) into Equation (6)

$$v_x dv_x = a_x dx \tag{8}$$

Integrate on both sides of Equation (8)

$$\frac{v_x^2}{2} = \int_0^x a_x dx \tag{9}$$

Integrate on both sides of Equation (7) and substitute Equation (9) into it

$$t = \int_0^t dt = \int_0^1 \frac{dx}{v_x} = \int_0^1 \frac{dx}{\sqrt{2\int_0^x a_x dx}} \tag{10}$$

Together with Equations (2) and (4), the integrations in Equation (10) can be carried out numerically through a computer program, where integration becomes summation and dx becomes a small interval Δx (e.g., 0.001).

$$t = \sum_{x=\Delta x/2}^{1} \frac{\Delta x}{\sqrt{2 \times \sum_{x=\Delta x/2}^{x} (a_x \Delta x)}} = \sum_{x=\Delta x/2}^{1} \frac{\Delta x}{\sqrt{2 \times \sum_{x=\Delta x/2}^{x} \left(g \frac{c_1 + 2c_2x + 3c_3x^2}{1 + \left(c_1 + 2c_2x + 3c_3x^2\right)^2} \Delta x \right)}} \tag{11}$$

Figure 3 shows a flow diagram of the numerical integration procedure.

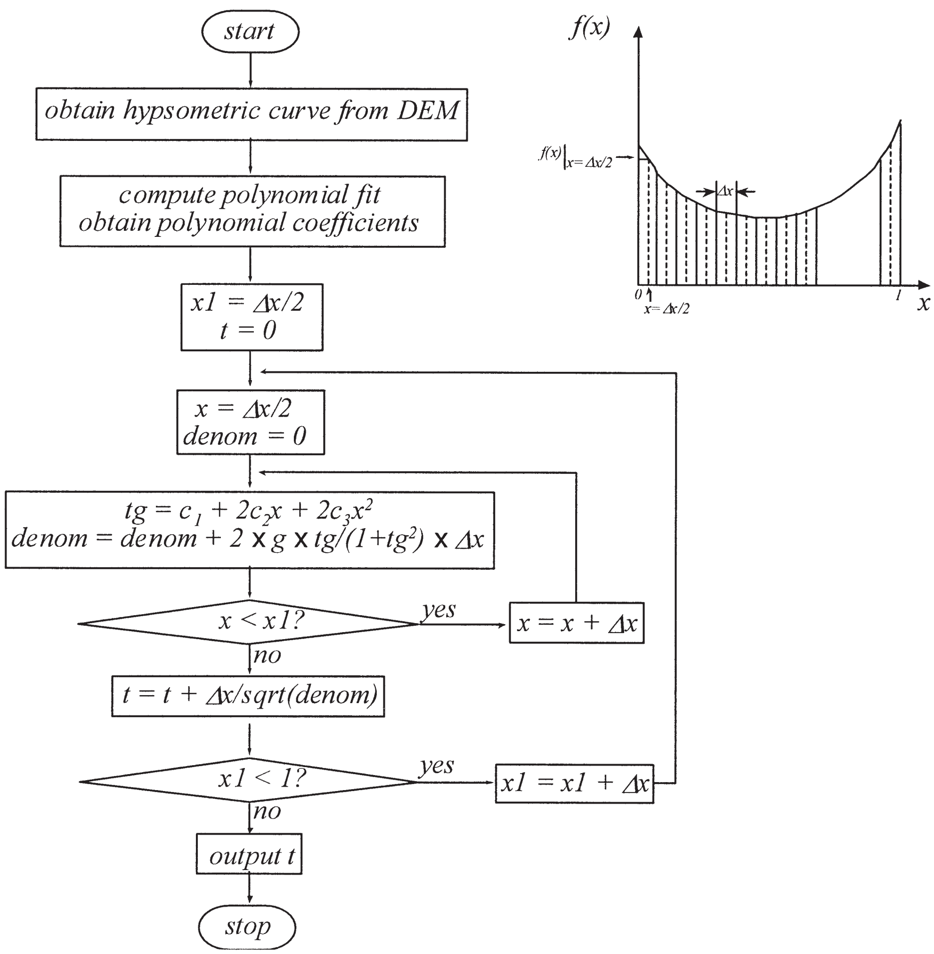

Figure 3. Flow Diagram Showing the Numerical Integration Procedure (g = gravitational acceleration constant (scaled by total relief); *tg, denom* = intermediate variables; t = travel time; and x = relative area).

PRELIMINARY APPLICATION

The above procedure was applied to 23 fluvial watershed basins over a broad range of climatic and geologic environments, including northern Alabama, central Oklahoma, and the driftless area in Iowa and Wisconsin (Harlin, 1982; Bridges, 1990; Luo, 1997, 2000). The hypsometric curves of the basins and the polynomial fits were obtained using an automated Geographic Information System (GIS) procedure (Luo, 1998, 2000) and the input data are the 7.5 minute Digital Elevation Model (DEM) downloaded from the

U.S. Geological Survey (USGS, 2002). Since the hypsometric curve normalizes relief to [0,1] by dividing elevation above the basin outlet by total relief (H in Figure 1), the gravitational acceleration constant is also scaled (divided) by the total relief (H in Figure 1) of each watershed basin. Similar to the empirical study that identified the relationship between time to hydrograph peak and hypsometric attributes (Harlin, 1978), the physically based theoretical travel time (dependent variable) is also regressed with the hypsometric attributes (independent variables, see the Appendix for definition of the attributes) using SAS (SAS Institute Inc., 1990). The hypsometric integral was excluded from the regression because it may not be unique (different hypsometric forms may have the same hypsometric integral) (Harlin, 1984). Table 1 shows the regressions between the theoretical travel time and all possible combinations of the hypsometric attributes. Most of the regressions are significant at the 0.0005 level with R^2 greater than 0.5. This result is also consistent with findings in Harlin (1984), which state that the observed time to hydrograph peak for a variety of watersheds is significantly correlated with their corresponding hypsometric attributes. Because the theoretical travel time derived here is conceptually analogous to the time to hydrograph peak, this result attests to the physical basis for the empirically established relationship between observed time to peak and hypsometric attributes (i.e., the hypsometric curve describes the gravity potential energy distribution throughout the watershed and that water moves down hill under the influence of gravity). It is also interesting to note that the most significant single variable regression is with hypsometric density. This is not surprising because the hypsometric density signifies the rate of change in overall basin slope.

The time of concentration is another popular parameter used to characterize watershed hydrologic response. It is often defined as the longest travel time a particle of water takes to reach the discharge point of a catchment (Wanielista *et al.,* 1997), which is conceptually similar to the theoretical travel time derived here. Thus the theoretical travel time is also compared with the time of concentration estimated by the Federal Aviation Administration method (FAA, 1970). The FAA method is shown in the following equation.

$$T_c = \frac{1.8(1.1 - C)\sqrt{L}}{\sqrt[3]{S}} \tag{12}$$

where T_c is time of concentration, L is the length of the longest travel path within the watershed, S is the slope of the watershed, and C is a constant depending on the character of the watershed.

TABLE 1. Regression Result Between the Theoretical Travel Time and Hypsometric Attributes.

Independent Variables				r^2	F-Value	p-Value
SK				0.0361	0.748	0.3973
KUR				0.0217	0.443	0.5133
DSK				0.5879	28.528	0.0001
DKUR				0.1534	3.624	0.0715
SK	KUR			0.6778	19.985	0.0001
SK	DSK			0.6293	16.129	0.0001
SK	DKUR			0.2319	2.867	0.0816
KUR	DSK			0.6397	16.866	0.0001
KUR	DKUR			0.5039	9.649	0.0013
DSK	DKUR			0.5968	14.060	0.0002
SK	KUR	DSK		0.6779	12.627	0.0001
SK	KUR	DKUR		0.6965	13.772	0.0001
SK	DSK	DKUR		0.6436	10.837	0.0003
KUR	DSK	DKUR		0.6630	11.806	0.0002
SK	KUR	DSK	DKUR	0.6970	9.774	0.0003

Notes (see Appendix for definition of these parameters):
SK = skewness of the hypsometric curve (or hypsometric skewness.
KUR = kurtosis of the hypsometric curve (or hypsometric kurtosis).
DSK = skewness of the hypsometric density function (or density skewness).
DKUR= kurtosis of the hypsometric density function (or density kurtosis)

Of the many empirical equations for estimating time of concentration (e.g., summarized in Viessman and Lewis, 1995, p. 183), the FAA method was selected because: (1) it is primarily designed for overland flow in urban watersheds and airfields, which have impermeable surfaces, consistent with the theoretical calculation assumptions used here, and (2) it only depends on the slope and flow length, both of which can be easily obtained from DEM with a GIS. In calculating the time of concentration for the 23 selected watersheds, C = 0.95 (the value for concrete surface) was used (to be consistent with impermeable surface assumption). The results are shown in Figure 4. Although there is scattering, the correlation is significant with a p-value of 0.0001, F-value of 26.88, and R^2 of 0.5614. The theoretical travel time is generally consistent with the empirically estimated time of concentration. This result further confirms the validity of the approach and also offers a potential practical use of this method.

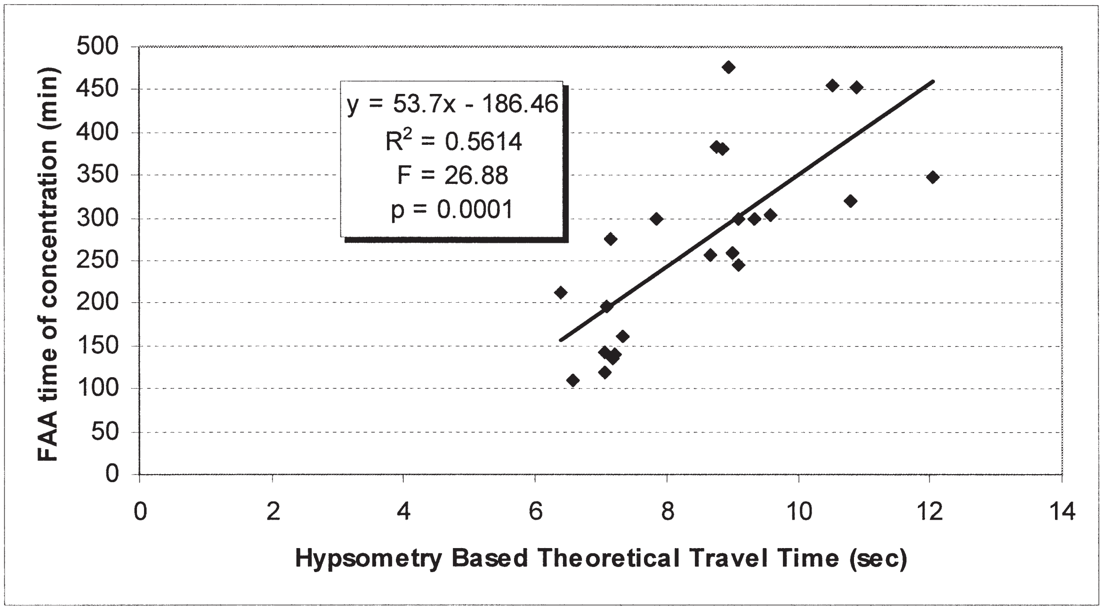

Figure 4. Correlation Between the Theoretical Travel Time and Empirically Derived Time of Concentration Using the FAA Method (Equation 12).

DISCUSSION AND CONCLUDING REMARKS

Although the hypsometric curve cannot be directly equated to the physical ground surface slope of a watershed, Strahler (1952) did show a linear relationship between the two if the slope of the hypsometric curve is adjusted for the contour length. However, the authors chose not to adjust the hypsometric slope to physical ground slope with contour length, because the travel time developed here is meant to be a theoretical time rather than an actual time. Also, the hypsometric curve describes the landmass distribution and thus the potential energy distribution within the basin above its base. The relationship will become less clear if the hypsometric slope is adjusted.

The derivation of the theoretical travel time is simply based on physical laws and the hypsometric curve derived from basin topography. In addition, friction was ignored in the present calculation. Thus, the theoretical travel time calculated is by no means equal to the real measured time to hydrograph peak. This travel time alone does not provide a full description of the hydrologic response at the watershed scale. Thus, it is not directly comparable with those physically based hydrologic response models such as TOPMODEL (Beven and Kirkby, 1979). However, this theoretical travel time does offer an alternative and simple way of measuring the effect of basin morphometry on flood response, which would allow comparison of such effect among different basins. The only data required is topography. In addition, this measure has the potential to be combined with other factors (such as soil characteristics, vegetation cover, rainfall intensity, land use, preexisting wetness conditions, surface roughness, etc.) to improve overall modeling accuracy of hydrologic response.

Based on simple physical laws, this paper demonstrates a physical basis for the linkage between hypsometric curve and hydrologic response, which was first identified through empirical data (Harlin, 1984). This linkage exists because the geomorphic characteristics of a watershed determine the potential energy distribution throughout the watershed, which can be quantitatively described by the hypsometric attributes, if the hypsometric curve is treated as a cumulative probability distribution.

With the GIS, digital data, and computing techniques that are readily available, theoretical travel time derived here offers an alternative way of characterizing the influence of basin morphometry on flood response, which is a critical component for developing an accurate flood potential index for flash flood warning (Sweeney *et al.,* 1993; Carpenter *et al.,* 1999). Furthermore, this study suggests that hypsometric curve and its statistical attributes may have great potential in improving predicting power if incorporated explicitly in hydrological modeling.

APPENDIX

The statistical attributes in the hypsometric technique include the hypsometric integral (INT); skewness of the hypsometric curve (hypsometric skewness, SK); kurtosis of the hypsometric curve (hypsometric kurtosis, KUR); skewness of the hypsometric density function (density skewness, DSK); and kurtosis of the hypsometric density function (density kurtosis, DKUR). The definitions of each term used in this paper were originally given in Harlin (1978) and are reproduced here for easy reference.

$$\text{Hypsometric Integral} = \text{INT} = \iint_R dxdy \quad \text{(A1)}$$

where R = the region under the hypsometric curve, x = relative area, and y = relative height.

$$\text{Hypsometric Skewness} = \text{SK} = \frac{\mu_{30}}{\left(\mu_{20}{}^{1/2}\right)^3} \quad \text{(A2)}$$

where μ_{30} = the third-order moment about x.

$$\mu_{30} = \frac{1}{INT}\iint\left(x - \mu_{10}\right)^3 dydx \quad \text{(A3)}$$

where μ_{10} is the first-order moment or x-mean or x-centroid.

$$\mu_{10} = \frac{1}{INT}\iint_R xdydx \quad \text{(A4)}$$

where μ_{20} = the second moment about x or variance.

$$\mu_{29} = \frac{1}{INT}\iint\left(x - \mu_{10}\right)^2 dydx \quad \text{(A5)}$$

$$\text{Hypsometric Kurtosis} = \text{KUR} = \frac{\mu_{40}}{\left(\mu_{20}{}^{1/2}\right)^4} \quad \text{(A6)}$$

where μ_{40} = the fourth-order moment about x.

$$\mu_{40} = \frac{1}{INT}\iint\left(x - \mu_{10}\right)^4 dydx \quad \text{(A7)}$$

Density skewness (DSK) and density kurtosis (DKUR) are defined similarly except that y is the first derivative of the hypsometric curve (i.e., the density function of the hypsometric curve; replacing y with y´).

ACKNOWLEDGMENTS

We would like to thank Warren Campbell for his helpful comments and the reviewers for their helpful reviews.

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