Financial Contracting, Signal-Jamming, and Entry Deterrence

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Abstract

We study the relationship between financial contracting and entry deterrence when the potential entrant observes the market price but does not observe the financial contract. This leads to the possibility that the entrant and the lender have different beliefs about the incumbent’s costs, due to uncertainty in the demand for the good. We show that as a result, the incumbent produces a different level of output in the first period and the probability of entry increases compared to the case when the entrant observes the financial contract.

Keywords: Experimentation, Strategic Experimentation, Signal Dampening, Signal Jamming, Financial Intermediation, Entry Deterrence

JEL classifications: C73 (Stochastic and Dynamic Games), D8 (Information and Uncertainty), L1 (Market Structure, Firm Strategy, and Market Performance)
1 Introduction

Economic decisions are taken in the context of a given informational environment. These decisions cannot form a consistent equilibrium unless all agents take account of the informational content of all market variables. In this process the beliefs of the agents play a central role, especially in the formation of the (informational) equilibrium. Indeed, it is not only the informational context of the actions of the agents that play an important role but also the beliefs and expectations embedded in these actions that affect the equilibrium. Understanding the effect of beliefs on the equilibrium is, thus, an important ingredient in understanding the implications implicit in equilibrium outcomes.

In this paper, we study the effect of information and expectations on the optimal (separating) contracts in a dynamic principal agent model, in the context of financial contracting and entry-deterrence. Specifically, we study the effect of potential entry on the contract between a bank and its client (an incumbent firm), assuming that the incumbent knows its cost (either high or low cost) but the bank does not and that demand is uncertain. The incumbent borrows from the bank in each of the two periods. It is assumed that the incumbent faces a threat of entry in the second period. Hence, information is important to both the bank and the potential entrant. Important issues, in this context, are how the information is generated and what information is observed. There are two crucial assumptions of our model that affect these issues. The first is
that the bank cannot directly observe the output of the incumbent but can only see a signal of it, i.e., the (random) price of the good. Bayesian updating is employed to update beliefs about the incumbent’s type. Secondly, the contract is hidden, that is, it cannot be observed directly by the entrant. If the entrant can observe the contract between the bank and the incumbent, then the entrant can deduce the incumbent’s output. However, since the contract cannot be observed the output of the incumbent must be postulated by the entrant. In this case, the bank may change the outputs designed for the incumbent, i.e., signal-jam. In other words, the bank signal-jams in order to manipulate the output of each type of incumbent, to take advantage of the entrant’s fixed beliefs. In order to study this process, we must consider how the optimal contract is designed when the bank’s and the entrant’s updated beliefs diverge, although in equilibrium they must coincide.

The purpose of this paper is to study the effect of signal jamming on outputs and entry, as well as on the financial contracts in the two periods. We provide comparisons with the case when the financial contract is public, that is, observed by the entrant. The model employed in this paper is based on Jeitschko, Mirman and Salgueiro (JMS), 2001 and Jeitschko and Mirman (JM), 2001, which study dynamic contracts under uncertainty.

The entry-deterrence game in this paper is similar to Milgrom and Roberts (MR), 1982 except that we add uncertainty as in Matthews and
Mirman (MM), 1983. Moreover, in contrast to MR and MM, we explicitly model the second period game, the outcome of which depends on the second period, or posterior, beliefs. Modelling the second period game requires that we study the informational effects of the first period decisions on the second period outcomes. Indeed, the effect of signal jamming can only be studied if the second period game is modelled explicitly. Moreover, in the agency context that we employ, the explicit second period game has an interesting characteristic. The agency game actually simplifies the entry rule since, given entry, the incumbent’s expected output when costs are not revealed to the bank, is equal to the high cost firm’s best response. This makes entry profitable whenever the entrant believes that the incumbent does not have low costs. That is, conditioned on entry, the entrant’s output decision is independent of the beliefs, thus, simplifying the entry rule. This is in contrast to the models of MR and MM if the second period game is modelled. Indeed, there the entrant’s output conditioned on entry would depend on his beliefs.

This paper also contributes to the growing literature on the relationship between the real and financial decisions of a firm, (see Brander and Lewis, 1986, Bolton and Scharfstein, 1990, Maksimovic, 1988 and Maksimovic and Titman, 1991.) and the effect of information on these decisions (see Gertner, Gibbons and Scharfstein, 1988 and Poitevin, 1989, 1990 and Faure-Grimaud, 1997, 2000). While the existing literature dealing with the relationship between financial contracting and product mar-
ket competition share the spirit and several features of our model, none of these papers integrates explicit modelling of prices and outputs in a dynamic model under uncertainty with an agency relationship. Further, the role of signal jamming, while studied in other contexts (see Jain and Mirman, 2001, Mirman, Samuelson and Urbano, 1993 and Fudenberg and Tirole, 1986) has not been addressed in the context of an integrated model of agency and the entry–deterrence game.

This work follows along the same lines as two of our previous papers, Jain, Jeitschko and Mirman (2002) and (2003), henceforth JJM1 and JJM2. In JJM2 the question of the effects of expectations on the equilibrium contracts in a dynamic principal agent model is studied under the assumption that demand is non–stochastic and thus there is complete learning in a separating contract. While in JJM2 the first period contract is also not observed by the entrant, the deterministic demand implies, as in MM, that the entrant’s beliefs dictate which equilibrium contract occurs. Indeed, when the contract is public in JJM2, the unique, optimal separating contract, which implies no limit pricing by the incumbent since the outputs remain the same with or without entry, is the only equilibrium contract. In contrast, when the contract is not public there is a continuum of equilibrium contracts—including the one with no limit pricing—depending upon the entrant’s beliefs. JJM1 introduce uncertainty in the JJM2 model and also assume that the contract is publicly observed. This assumption implies that the posterior of the bank and the
entrant must be the same since they observe the same variables and the entrant can deduce the incumbent's output. A significant effect of uncertainty is the possibility of incomplete learning even under a separating contract. This in turn leads to a trade-off between learning to increase expected profits in the future and not learning to keep the benefits from mimicking low, i.e., the signal dampening effect.

In this paper we deal with an intermediate case. We consider the situation in which outputs are not observable, i.e., prices are a noisy signal of outputs—as in JJM1—and the contract is not public—as in JJM2. Neither the bank nor the entrant can observe the incumbent’s output, and thus cannot know for sure the incumbent’s type. However, in this paper, since the entrant cannot observe the contract, the outputs specified by the bank for the incumbent cannot be deduced by the entrant. Thus, the entrant must make conjectures about the incumbent's output. The entrant uses Bayes' rule to update its beliefs about the type of firm it faces. In this paper, the presence of uncertainty combined with unobservability of the first-period contract allows us to study the informational effects of the interaction between the agency game and the entry-deterrence game. In particular, we study the informational effect on the real outputs that the bank chooses for the two possible incumbent types as well as the effect on the probability of entry.

We find that uncertainty in the demand function and thus the possibility of incomplete learning in the second period leads to important
changes in the financial contract as well as in the outcomes of the entry
deterrence game. First, the interaction of a principal-agent framework
with the entry-deterrence game under uncertainty implies that contrary
to the standard results of an agency model, the low cost incumbent no
longer produces the first best output. That is, the distortion induced in
the high cost incumbent’s output and the feedback effect from the po-
tential entrant changes the low cost incumbent’s best response as well.
In other words, uncertainty and the possibility of a Cournot game in the
second period implies that the output choices of the two types of incum-

Second, the possibility of incomplete learning implies that the low
cost incumbent has less of an incentive to mimic the high cost incumbent
in the first period (see JMS and JM), since noise reduces the probability
that the low cost incumbent is perceived as the high cost incumbent.
Thus, as compared to the static case, the bank has an incentive to set
the outputs closer together, reducing the informational content of the
price observation. The threat of entry in the second period weakens this
incentive, since entry does not occur when the incumbent is known to
be low-cost. Hence, reducing the incumbent’s incentive to deceive the
bank. That is, the bank’s incentive to set outputs closer together, due
to signal dampening, is less powerful with entry than without. On the
other hand, experimentation, that is, the bank’s incentive to learn about
the incumbent’s type and thus increase its second period profits, takes
on a strategic form due to the threat of entry and is strengthened due to lower profits if entry occurs.\footnote{This strategic experimentation is different from Bolton and Harris, 1999, though, where strategic experimentation appears in the context of a population game.}

Thus, uncertainty and potential entry reinforce each other. Without entry the experimentation and signal dampening effects result in first-period outputs that are set closer together then in the static case, so that the signal dampening effect is dominant. The overall effect on the first period contract of potential entry is to increase the informativeness of the first-period price and thus deter entry with a higher probability.

We also find that the observability of the contract matters. In particular, as in JJM2, there is a continuum of equilibrium contracts, since the entrant cannot observe the contract. Hence, the equilibrium contract is dependent on the entrant’s postulated beliefs. Moreover, in all cases the first period equilibrium contract, when it is not observable, implies a higher output for the high cost incumbent and the same output for the low cost incumbent, compared to the public contract case. This, in turn, leads to a higher probability of entry compared to the public contract case. Thus, signal jamming occurs and the inability of the bank to commit via a public contract and its incentive to deter entry lead to ‘more’ entry. Underlying this result is the fact that when the contract is not observed by the potential entrant, while the signal dampening effect remains unaffected, the experimentation effect is weakened in all cases of expectations consistent with equilibrium. That is, the bank is less ef-
fective in deterring entry and thus learning via the first period outputs when the entrant cannot observe the contract. Thus, the first period outputs are set closer together due to signal jamming and the probability of entry increases. In other words, signal jamming is costly.

2 The Model

There are two time periods. Inverse demand for the good in each period is given by \( p = a - bq + \epsilon \), where \( a \) and \( b \) are known parameters and \( \epsilon \) is a random, unobservable term that is distributed uniformly on the interval \([-\eta, \eta]\) and is independent across the two periods. Firms supply the quantity \( q \) and thereafter \( p \), hence the price, is realized. The price \( p \) is publicly observable, but each firm’s output is private information of the firm and unverifiable.

In the first period, only the incumbent firm is in the market. In order to produce, it borrows (an amount, for simplicity, normalized to 0) from a financial intermediary (i.e., a bank). The relationship between the incumbent and the bank is modeled as a principal-agent relationship. The bank offers a take-it-or-leave-it repayment schedule that maps the observed (and hence verifiable) market price \( p \) into an amount \( R(p) \) that the incumbent must pay to the bank. Both the incumbent and the bank are risk-neutral expected profit maximizers with unlimited liability. Due to risk-neutrality of the players, there are an infinite number of repayment schemes that implement the equilibrium outputs at equally low cost for
the bank. We present only the real implications of the equilibrium contract, i.e., the equilibrium outputs, expected profits, and expected payoffs. In particular, we focus on expected repayments rather than the actual repayment schedule $R(p)$.

The technology used to provide the good implies a constant unit cost. There are two possible levels of the unit cost, denoted by $\tau$ and $c$, with $c < \tau < a$, the demand intercept. The incumbent knows the cost, the bank has beliefs $\rho$ that the incumbent’s cost is low, and $1 - \rho$ that the cost is high. The two “types” of incumbent are referred to as the low cost and high cost incumbent.

Taking the dynamic nature of the interaction into account, the bank’s objective is to maximize the sum of expected repayments from the incumbent in the two periods. Similarly, the incumbent’s objective is to maximize the difference between its expected gross profit and the expected repayment made to the bank. Let $\bar{R}$ and $\underline{R}$ denote the expected repayment by the high cost and the low cost incumbent, where the expectation is with respect to the distribution of $\epsilon$. The bank’s per-period expected payoff is represented as $u_b = \rho \bar{R} + (1 - \rho)\underline{R}$, whereas the firm’s per-period expected payoff is denoted by $\bar{u} = \bar{\pi} - \bar{R}$, where $\bar{\pi}$ is the expected gross profit of the high cost incumbent. Similarly, $\underline{u} = \underline{\pi} - \underline{R}$, for the low cost incumbent.

After the first period price becomes publicly observable, the incumbent makes a repayment to the bank in accordance with the scheme $R(p)$.

\footnote{See JMS for an example of how an equilibrium contract is derived.}
Since the price is publicly observable, it allows inferences about the firm’s output and thus about the incumbent’s cost. That is, the bank updates its beliefs about the incumbent’s type.

We assume that in the second period there is a potential entrant with the same prior beliefs about the incumbent’s type as the bank. The entrant has constant marginal cost of $c_e$ (assumed to be between $\overline{c}$ and $c$) and faces a fixed cost of entry denoted by $K$. The entrant is also a risk-neutral profit maximizer. For convenience, the entrant is assumed not to borrow. In order to determine whether to enter, the entrant too observes the market price and updates its beliefs about the incumbent’s cost. An important assumption of the model is that the entrant does not observe the financial contract.\(^3\) Consequently, the entrant must conjecture the incumbent’s first period output and then update its beliefs about the incumbent’s type on the basis of this conjecture in addition to the observed price.\(^4\) In this case, the bank accounts for the potential entrant’s ignorance of the actual contract and has the incentive to manipulate the contract in an attempt to `signal jam.’ Nevertheless—in equilibrium—the bank and the potential entrant must have the same be-

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\(^3\)However, it is important to make a comparison between the model under the assumption that the contract cannot be observed, i.e., when signal jamming is possible, and the case in which the contract is observable. The latter is the benchmark case. We make comparisons between these two cases throughout the paper.

\(^4\)This updating is very different from that studied in JJM2. Although in JJM2 the incumbent also conjectures the equilibrium outputs, because demand is deterministic, the observed price reveals the output.
liefs about the incumbent’s costs due to the potential entrant’s rational expectations about the bank’s incentives concerning the structure of the first period contract.

Based on the updated beliefs about the incumbent’s cost structure, the potential entrant decides whether or not to enter the market. This decision is made on the basis of the entrant’s expectation of profits, given that when there are two firms in the market these engage in simultaneous move quantity (Cournot) competition. Finally, the bank observes the entrant’s decision before designing an incentive scheme for the incumbent in the second period.

All players’ time preference is captured in the common discount factor $\delta$.

Although the environment is stochastic, the bounded support of the noise implies that the contract is not unique.$^5$ However, it is easily shown that the high cost incumbent always produces less than a low cost firm and this fact is common knowledge. Given this property it is then natural to consider only beliefs that are monotonic. That is, a belief that the incumbent is the low type based on some price observation is no larger for any observation of any higher price. This assumption also addresses out-of-equilibrium price observations.$^6$

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$^5$Unlike deterministic models, in modelling a stochastic environment, when noise has full support, a unique equilibrium usually emerges, see e.g., the contrast between MR, with many equilibrium points in a deterministic model and MM, with a unique equilibrium in a similar but stochastic model.

$^6$It is shown below that this does not yield uniqueness of the equilibrium, but it does
We examine the effect of the unobservability of the financial contract on prices and outputs. In particular, we ask if signal jamming occurs and how the equilibrium contract with signal jamming differs from the equilibrium contract if the entrant were to observe the contract before making an entry decision.

The equilibrium is a perfect Bayesian equilibrium. Thus, all beliefs about agents’ types and actions are mutually consistent with the actions taken in equilibrium, including beliefs about actions taken in accordance with which contracts are in place between the bank and the incumbent firm. Moreover, equilibrium beliefs are updated using Bayes’ rule.

Finally, we make two assumptions that assure that both types of incumbent produce a positive amount of output and that the first period price does not necessarily reveal the incumbent’s type.

**Assumption 1** *The market is sufficiently profitable; that is, the demand intercept $a$ is large.*

**Assumption 2** *Given outputs, the range of possible prices is large; that is, $\eta$ is large.*

As demonstrated in JMS, an implication of Assumptions 1 and 2 is that the equilibrium repayment scheme in both periods has the low cost incumbent’s incentive compatibility and the high cost incumbent’s individual rationality constraints binding—all other constraints are slack. Limit the equilibrium outcomes to a compact connected set.
3 The Second Period

In this section the second period contract is derived. Although our objective is to study the hidden contract and signal jamming, the second period implications of the public contract are also derived, in order to study the relationship between these two contracts. These two assumptions on the observability of the contract not only yield different results but require a different approach. In order to find the effect of signal jamming it is necessary to give the bank the ability to evaluate the effect of choosing contracts, i.e., first period outputs for the two types of the incumbent, that are different from those expected by the entrant. Recall, in any rational expectations equilibrium, the beliefs of the bank and the entrant must coincide, even when the entrant cannot actually observe the contract between the bank and the incumbent. However, since the contract is not observable, the entrant must make all decisions on the basis of subjective expectations of the outputs in the contract. The bank is able to manipulate the contract, thus taking advantage of the entrant’s expectations. Consequently, the question arises whether the equilibrium contract and the first period equilibrium outputs change due to the unobservability of the contract. We find that signal jamming occurs. Indeed, the first-period outputs are set closer together when the contract is hidden to the entrant, leading to a higher probability of entry compared to the public contract.

Two steps are necessary in formalizing and evaluating the effect of
signal jamming in the current model. First, it must be determined how manipulating the first period contract, i.e., manipulating the outputs, affects the distribution of first period prices and thus the probability of entry. Second, the potential benefits of such manipulation must be quantified.

In order to determine how manipulating the first period outputs affect the probability of entry, one must consider the possibility that the entrant has different beliefs about the first period outputs than those that the bank actually specifies. Let $q_e$ and $Q_e$ denote the potential entrant’s conjecture about the outputs specified in the first period contract. Similarly, denote by $q^i$ and $Q^i$ the first period outputs that the two types of incumbent actually are induced to produce.

Given these (possibly different) beliefs about the first period outputs of the two types of incumbent, the bank and the entrant draw (possibly different) inferences about the incumbent’s type on the basis of the observed first period price. To distinguish these, let the bank’s posterior, i.e., the objective posterior probability regarding the incumbent firm’s type based on the actual first period incentive contract, be denoted by $\rho_2$, whereas the potential entrant’s subjective belief based on his conjecture about first period outputs is denoted by $\rho^e_2$.

In order to derive the equilibrium, we first consider the implications for the bank when choosing first period outputs that are larger than those conjectured by the potential entrant. That is, for given $q^e$ and $Q^e$, we
study the implications for the bank when choosing \( q^i > q^e \) and \( \overline{q}^i > \overline{q}^e \).

Since the noise is uniformly distributed, the posterior of the bank and the entrant can be easily derived and is given in the following lemma. Note that the lemma includes out-of-equilibrium beliefs which have been chosen to mimic noise on the entire real line with the monotone likelihood ratio property.\(^7\)

**Lemma 1 (Posterior Beliefs, Increased Outputs)** The posterior of the bank and the potential entrant, when the contract is hidden and the bank chooses first period outputs such that \( \overline{q}^i > \overline{q}^e \) and \( q^i > q^e \), is given by,

\[
(r_2, r_2^e) = \begin{cases} 
(1, 1), & \text{if } p_1 \in (-\infty, a - b\overline{q}^i - \eta), \\
(p, 1), & \text{if } p_1 \in [a - b\overline{q}^i - \eta, a - b\overline{q}^e - \eta), \\
(p, p), & \text{if } p_1 \in [a - b\overline{q}^e - \eta, a - bq^i + \eta), \\
(0, p), & \text{if } p_1 \in [a - bq^i + \eta, a - bq^e + \eta), \\
(0, 0), & \text{if } p_1 \in [a - bq^e + \eta, +\infty). 
\end{cases}
\]

The ex ante probabilities of the five different states in the second period

\(^7\)In considering the bank's incentives to change outputs on the margins it is innocuous to postulate that \( \overline{q}^i < q^e \). Assumption 2 then ensures that the intervals have positive length and the associated probabilities are positive.
are,

\[
\begin{align*}
\Pr\{\rho_2, \rho_2^e = (1, 1)\} &= \rho \frac{b(q^i - \pi^i)}{2\eta}, \\
\Pr\{\rho_2, \rho_2^e = (\rho, 1)\} &= \frac{b(q^i - \pi^i)}{2\eta}, \\
\Pr\{\rho_2, \rho_2^e = (\rho, \rho)\} &= \frac{2\eta - b(q^i - \pi^i)}{2\eta}, \\
\Pr\{\rho_2, \rho_2^e = (0, \rho)\} &= (1 - \rho) \frac{b(q^i - \pi^i)}{2\eta}, \\
\Pr\{\rho_2, \rho_2^e = (0, 0)\} &= (1 - \rho) \frac{b(q^e - \pi^i)}{2\eta}.
\end{align*}
\]

To provide a benchmark for the results of signal jamming, the following lemma gives the posterior for the case of the public contract.

**Lemma 2 (Posterior Beliefs, Public Contract)** The posterior of the bank and the potential entrant, when the contract is public, is given by,

\[
\rho_2 = \begin{cases} 
1, & \text{if } p_1 \in (-\infty, a - bq^i - \eta), \\
\rho, & \text{if } p_1 \in [a - bq^i - \eta, a - bq^i + \eta), \\
0, & \text{if } p_1 \in [a - bq^i + \eta, +\infty).
\end{cases}
\]

The ex ante probabilities of the three different states in the second period are,

\[
\begin{align*}
\Pr\{\rho_2 = 1\} &= \rho \frac{b(q^i - \pi^i)}{2\eta}, \\
\Pr\{\rho_2 = \rho\} &= \frac{2\eta - b(q^i - \pi^i)}{2\eta}, \\
\Pr\{\rho_2 = 0\} &= (1 - \rho) \frac{b(q^e - \pi^i)}{2\eta}.
\end{align*}
\]

Note that either the incumbent’s type is completely revealed to the bank and to the entrant, or the posterior of the bank and the entrant remains the same as the prior—an implication of the uniform distribution
of the noise. Also notice that, if \( q^i = q^e \), then \( \rho^2 = \rho^e \) and, for a given first period contract, the two Lemmas coincide.

In order to evaluate the bank’s second period payoffs corresponding to the five distinct constellations of beliefs in the second period, we need to determine the entry rule. The assumption below is made for concreteness and is consistent with MR.

**Assumption 3** The entrant’s fixed cost of entry \( K \) is such that entry is not profitable when the incumbent firm has low cost and entry is profitable when the incumbent has high cost.

An important feature of the model, as demonstrated below (in Proposition 1), is that given entry, the incumbent’s expected best response function when the type is not revealed is equal to that of a high cost incumbent firm. Consequently, since the entrant is also risk neutral, the entry rule for the potential entrant has the feature that it specifies the same action concerning entry whether the posterior is equal to \( \rho \) or 0. In other words, although the entrant’s beliefs take on three possible values, the entry rule only distinguishes between two cases. Either the incumbent is believed to have low cost \( (\rho^e_2 = 1) \) and entry does not occur, or the incumbent may have high cost \( (\rho^e_2 = \rho, 0) \), in which case entry does occur. Formally, letting 1 denote that entry occurs and 0 that entry does not occur, the entry rule is then given by,
Lemma 3 (Entry Rule) The entrant’s decision rule is given by

\[
e = \begin{cases} 
1 & \text{if } \rho_2^x = 0, \rho \\
0 & \text{if } \rho_2^x = 1.
\end{cases}
\]

We next calculate the bank’s second-period profits corresponding to the five different belief-pairs and compare these with the payoffs in the public contract. We start with the belief-pairs in which the entrant and the bank have the same posterior so that the resulting profits are the same as in the public contract.

First, suppose \((\rho_2, \rho_2^x) = (1, 1)\), that is, both the bank and entrant believe that the incumbent has low cost. By Lemma 3, the entrant does not enter. The second period equilibrium contract is the same as the static full-information contract in which the bank demands as repayment the entire monopoly profit of the incumbent. Thus, the bank’s profits corresponding to \((\rho_2, \rho_2^x) = (1, 1)\) are,

\[
u_b = \frac{(a - c)^2}{4b}.
\] (1)

There are no informational rents, so that the incumbent is left with a payoff of zero, i.e.,

\[u_i = \bar{u}_i = 0.\]

Notice that the equation includes a payoff for the high cost incumbent—even though, in equilibrium, whenever \(\rho_2 = 1\), the incumbent has low costs. This is because it is used to determine the high cost incumbent’s
incentives to deviate from the equilibrium in the first period. The high
cost incumbent’s payoff is 0, because this type would be unable to make
the repayment demanded by the bank.

Next, suppose that \((\rho_2, \rho_2^*) = (0, 0)\), so that both the entrant and the bank
believe that the incumbent has high cost. By Lemma 3, entry occurs
in this case. Given the bank’s belief that the incumbent has high cost,
the bank calculates the highest (i.e., “first best”) level of profit that the
high cost incumbent can obtain in Cournot competition with the entrant.
This is the second period repayment that the bank demands from the
incumbent.

Let \(q_i\) denote the output that the incumbent is expected to produce
in the second period. The incumbent’s expected profit function
\(\pi_i = (a - b (q_e + q_i) - \bar{c}) q_i\) yields a “reaction curve” of,

\[
q_i = \frac{(a - b q_e - \bar{c})}{2b}.
\]

Similarly, the entrant’s reaction curve is \(q_e = (a - b \bar{q}_i - c_e)/2b\). Given
the reaction curves, the equilibrium outputs are,

\[
\bar{q}_i = \frac{a + c_e - 2\bar{c}}{3b},
\]

\[
q_e = \frac{a + \bar{c} - 2c_e}{3b}.
\]

The expected profits that accrue to the entrant and the high cost in-
cumbent are,

\[
\pi_e = \frac{(a+\bar{c}-2c_e)^2}{9b} - K, \tag{3}
\]

\[
\pi_i = \frac{(a+c_e-2\bar{c})^2}{9b}.
\]
Hence, the bank demands \( R = \frac{(a + c_e - 2\tau)^2}{9b} \) from the incumbent, and induces the incumbent firm to produce \( q_i \). This yields the following expected net payoffs for the bank and the incumbent in the second period, when beliefs are given by \((\rho_2, \rho_2^e) = (0, 0)\),

\[
\begin{align*}
    u_b &= \frac{(a + c_e - 2\tau)^2}{9b}, \\
    u_i &= 0, \\
    u_i &= (\bar{c} - \zeta)\bar{q}_i.
\end{align*}
\]

(4) 
(5) 
(6)

The high cost incumbent—whose type is revealed—has no informational rent. Notice, however, the positive payoff to an incumbent firm that has low cost (Equation (6)). The payoff of a low cost incumbent is relevant even though, in equilibrium, whenever \( \rho_2 = 0 \), the incumbent has high costs, because the low cost incumbent’s first period incentives to deviate from the equilibrium must be assessed. Indeed, it is this positive payoff that may tempt a low cost incumbent to take an out-of-equilibrium action in the first period. This positive payoff is the second period value of deceiving the bank about its true costs when the incumbent has low cost. It is the cost advantage of the low cost incumbent over the high cost incumbent \((\bar{c} - \zeta)\) multiplied by the amount produced, \( \bar{q}_i \).

Next, suppose that \((\rho_2, \rho_2^e) = (\rho, \rho)\). In this case, both the bank and the entrant do not know the incumbent’s cost. As a consequence, an incentive scheme must be devised so that the incumbent is induced to produce the output that is best from the bank’s point of view, given the bank’s
uncertainty about the incumbent’s type. That is, in addition to consid-
ering participation constraints, the bank must also consider incentive
compatibility constraints.

Thus, letting $R$ denote the expected repayment of a low cost incum-

bent and $\bar{R}$ that of the high cost incumbent, the bank chooses expected
repayments in order to maximize,

$$u_b = \rho R + (1 - \rho)\bar{R}, \quad (7)$$

subject to the individual rationality constraints,

\[
\begin{align*}
\bar{R} &\leq (a - b(\bar{q}_i + q_e) - c)\bar{q}_i, \\
R &\geq (a - b(q_i + q_e) - c)q_i,
\end{align*}
\]

and the incentive compatibility constraints,

\[
\begin{align*}
(a - b(q_i + q_e) - c)q_i - R &\geq (a - b(\bar{q}_i + q_e) - c)\bar{q}_i - \bar{R}, \\
(a - b(\bar{q}_i + q_e) - c)\bar{q}_i - \bar{R} &\geq (a - b(q_i + q_e) - c)q_i - \bar{R}.
\end{align*}
\]

We now demonstrate that, conditioned on entry, the incumbent’s ex-
pected output is the same as the output produced when it is known that
the incumbent has high cost. Consequently, it is optimal for the potential
entrant to enter the market when $\rho_2^e = \rho$.

As is standard and straightforward to verify (only) the high cost in-
cumbent’s participation constraint binds in equilibrium. The firm’s reser-
vation utility is 0, so the high cost type expects to repay the entire profit.

$$\bar{R} = (a - b(\bar{q}_i + q_e) - c)\bar{q}_i. \quad (8)$$
Similarly, in equilibrium (only) the low cost firm's incentive compatibility constraint binds. Rearranging this binding constraint yields,

\[ R = (a - b(q_i + q_e) - c)q_i - (c - \epsilon)\bar{q}_i. \]  

(9)

Notice that the informational rent to the low cost incumbent is equal to the profit that a low cost firm would retain when producing the high cost incumbent’s output. Substituting the two binding constraints into the bank’s objective function, given in Equation (7), yields the bank’s maximization problem. The first order necessary and sufficient conditions of this problem yield the bank’s “reaction curves,”

\[ q_i = \frac{a - bq_e - c}{2b}, \]

\[ \bar{q}_i = \frac{a - bq_e - \tau}{2b} - \frac{\rho - \tau - \epsilon}{2b}. \]

The first-order condition of the entrant’s maximization problem yields the reaction curve,

\[ q_e = \frac{a - b(q_i + (1 - \rho)\bar{q}_i) - c_e}{2b} \equiv \frac{a - b\hat{q}_i - c_e}{2b}. \]

The following proposition shows that the reaction function of the unknown incumbent is the same as the reaction function of the high cost incumbent, given in Equation (2), and thus validates the entry rule in Lemma 3.

**PROPOSITION 1** The expected reaction curve of an incumbent of unknown type coincides with the reaction curve of an incumbent known to have high
cost. That is,

\[ \hat{q}_i \equiv \rho q_i + (1 - \rho) \overline{q}_i = \frac{a - b q_e - \overline{c}}{2b}. \tag{10} \]

The proof is straightforward and thus omitted.

The (type-dependent) equilibrium outputs are,

\[ q_e = \frac{a + \tau - 2c e}{3b}, \]
\[ q_i = \frac{a + c e - \tau - 2c e}{3b}, \]
\[ \overline{q}_i = \frac{a + c e - 2 \tau - \rho \tau - c e}{3b} \frac{b}{1 - \rho} + \overline{c}. \]

The next proposition shows that the outputs produced by the two types of incumbent in the second period are not first-best. That is, they are less than the outputs that would have been produced if there were no bank and thus no agency problem. Indeed, this result holds regardless of whether there is full information or incomplete information in the Cournot game.

**Proposition 2** Compared to "first best" outputs produced in standard Cournot competition of either full or incomplete information, the output of either type of incumbent is smaller when the bank is present and there is an agency problem. That is,

\[ q_i < \frac{a + c e - \tau - 2c e}{3b}, \]
\[ \overline{q}_i < \frac{a + c e - 2 \tau - \rho \tau - c e}{3b}, \]

where

\[ \hat{c} = \rho \overline{c} + (1 - \rho) \tau. \]

---

\(^8\)Assumption 1 ensures that both types of incumbent produce a positive amount.
The proof is straightforward and thus omitted.\footnote{The Proposition holds even though, given the cost of entry \( K \), the potential entrant would not enter the market when knowing that the incumbent has low costs.}

In other words, the bank designs an incentive contract in which neither type of incumbent firm chooses the first best response of the standard Cournot duopoly game with or without complete information. This result is in contrast with the general static principal agent relationships in which the 'good' type, here the low cost incumbent, is induced to take a 'first-best' action. The intuition for this Proposition is that in contrast to a typical static agency problem, here the agent (the incumbent) is engaged in a game with a third party (the entrant). This integration of the agency problem with the Cournot game has the effect of creating interdependence between the decisions of all the types of players. In particular, the low cost incumbent's binding incentive compatibility constraint forces a distortion of the high cost incumbent's output away from (below) its first best level. The entrant does not know what type of firm the incumbent is and responds in part to the high cost incumbent's distorted output, thus, leading to a different output for the low cost incumbent than when there is no bank. That is, the low cost incumbent's output is a response to the entrant's altered output.

Put another way, although the low cost incumbent's reaction curve is the standard 'first-best' response function, that of the high cost incumbent is not. This distortion (induced by the low cost incumbent's binding incentive compatibility constraint) affects the entrant's output and this,
in turn, induces the low cost incumbent to produce less than the best response in a standard Cournot duopoly.

Despite the fact that neither type of incumbent produces the best response in a Cournot game between the incumbent and entrant, the incumbent’s expected output is equal to the first-best output of the high cost incumbent.

**Proposition 3** Conditioned on entry, both firms’ (the incumbent’s and the entrant’s) expected equilibrium outputs are the first best responses from the Bayesian Cournot Nash equilibrium of the standard Cournot duopoly game in which the incumbent is known to have high cost. Thus, for $\rho_2$ equal to either $\rho$ or 0, the expected outputs are

\[
q_e = \frac{a + c - 2c_e}{3b},
\]

\[
\hat{q}_i \equiv \rho_2 q_i + (1 - \rho_2) \bar{q}_i = \frac{a + c - 2c}{3b}.
\]

The result follows immediately from the equilibrium output levels derived above and is seen in Figure 1, where the incumbent’s “expected reaction curve” coincides with the “first best” reaction curve of a high cost firm.
Figure 1: The incumbent’s reaction curves from the standard Cournot duopoly game (thin steep lines) and from the game altered by the bank’s presents (thick steep lines). The curve furthest from the origin is the one of the low cost incumbent, the one’s closest to the origin, those of the high cost incumbent. The dashed line represents the “expected reaction curve” from the viewpoint of the potential entrant facing an incumbent of unknown costs.

Compared to the standard Cournot game (thin steep lines) the bank shifts the high cost incumbent’s reaction curves to smaller levels of output (thick steep line close to origin). Consequently the incumbent’s “expected reaction curve” (thick dashed line) coincides with the “first best” reaction curve of a high cost incumbent with out a bank.

The flat line is the potential entrant’s reaction curve.

**Corollary** Conditioned on entry, the entrant’s output is independent of the posterior beliefs $\rho_2^e$ concerning the incumbent’s cost.

Indeed, once the potential entrant has entered the market the output decision is independent of the value of the prior, $\rho$. Another immediate consequence of Proposition 3 and its Corollary is that, given entry, the entrant’s expected profit is the same whether or not the entrant believes to have inferred the incumbent’s type. That is, for $\rho_2^e = 0$ and $\rho_2^e = \rho$ alike, the entrant’s expected profit is given in Equation (3). For the
incumbent, the expected profit is,

\[ \Pi_i = \left( \frac{a + c_e - \frac{c}{2}}{y_b} \right)^2, \]

\[ \Pi_i = \left( \frac{a + c_e - 2\bar{\tau}}{y_b} \right)^2 - \left( \frac{\rho}{1 - \rho} \right)^2 \frac{(\bar{\tau} - \bar{c})^2}{4b}. \]

Substituting the expected outputs into the binding constraints (8) and (9), some algebraic manipulation, yields

\[ R = \left( \frac{a + c_e - 2\bar{\tau}}{y_b} \right)^2 + \frac{1 + \rho}{1 - \rho} \frac{(\bar{\tau} - \bar{c})^2}{4b}, \]

\[ \bar{R} = \left( \frac{a + c_e - 2\bar{\tau}}{y_b} \right)^2 - \left( \frac{\rho}{1 - \rho} \right)^2 \frac{(\bar{\tau} - \bar{c})^2}{4b}. \]

The incumbent’s and the bank’s payoffs in the second period when the incumbent’s cost remains unknown to both the bank and the entrant, i.e., \((\rho_2, \rho_2^e) = (\rho, \rho)\), are thus,

\[ u_b = \frac{(a + c_e - 2\bar{\tau})^2}{y_b} + \frac{\rho}{1 - \rho} \frac{(\bar{\tau} - \bar{c})^2}{4b}, \quad (11) \]

\[ \mu_i = 0, \]

\[ u_i = (\bar{\tau} - \bar{c})\bar{q}_i. \]

Notice that although the expected revenue is the same as when the incumbent is known to have high cost (an implication of Proposition 3), the bank actually has a higher payoff than in that case (compare Equation (11) with Equation (4)), since now there is a positive probability that the incumbent actually has low costs. The resulting increase in expected profit is shared between the bank (as the principal) and the incumbent firm (yielding the ‘good’ agent’s information rent).
The three second period scenarios analyzed thus far have the property that the bank and the entrant have the same posterior despite the fact that the entrant does not observe the financial contract. JJM1 show that if the entrant observes the financial contract, that is, if the financial contract is public, these three are the only scenarios to consider. In other words, in these three cases the public and private contract yield the same outputs and therefore, the same profits. The difference, due to signal jamming, comes from the following two cases, in which there is divergence between the actual and expected outputs.

We now consider the two cases in which \( \rho_2 \neq \rho_2^e \), that is, the cases in which the signal (the observable first period price) leads to differing posterior beliefs. When \((\rho_2, \rho_2^e) = (\rho, 1)\), the potential entrant believes the incumbent firm to be the low type, but it may not be. Consequently, entry is deterred despite the fact that the bank is not sure about the incumbent’s type. In this situation, given the bank’s uncertainty about the incumbent’s type, it devises an incentive scheme for the incumbent firm as the sole firm in the market in the second period. This incentive scheme is obtained by setting the entrant’s output, \( q_e \), equal to zero in the contract derived earlier for the case when both the bank and the entrant are unsure about the incumbent’s type and entry occurs.

Specifically,

\[
q = \frac{a - c}{2\varphi},
\]

\[
\bar{q} = \frac{a - \overline{c}}{2\varphi} - \frac{\rho}{1 - \rho} \frac{\overline{c} - e}{2\varphi}.
\]
So that the payoffs are,

\[ u_b = \frac{(a - \tau)^2}{4b} + \frac{\rho}{1-\rho} \frac{(\tau - c)^2}{4b}, \]

\[ \bar{u} = (\bar{c} - \zeta)q, \]

\[ \bar{u} = 0. \]

Finally, consider the case in which \((\rho_2, \rho_2^e) = (0, \rho)\). The bank knows that the incumbent is a high cost firm, but the entrant is uncertain. Recall from the Corollary, however, that conditioned on entry, the entrant’s output is independent of the subjective beliefs. Consequently, this case is identical to the case where both know that the incumbent has high cost. Thus, the payoffs are given as before (see Equations (4)-(6)) by,

\[ u_b = \frac{(a + c + 2\tau)^2}{9b}, \]

\[ \bar{u_i} = 0, \]

\[ \bar{u}_i = (\bar{c} - \zeta)q_i. \]

The analysis undertaken thus far relates to a specific scenario concerning the bank’s choice of first period outputs given the entrant’s conjecture of these outputs, namely one for which, \(q^i > \bar{q}^i\) and \(q^e > q^e\). However, the bank may also want to consider decreasing the first period outputs, given the entrant’s conjecture. That is, given the entrant’s conjecture about the first period contract, the bank may choose outputs such that,
$q^i < q^e$ and $q^l < q^e$. An implication of such divergence is that the entrant associates lower prices with the high cost incumbent than implied by the actual outputs. In this case, the entrant’s posterior changes as follows:

**Lemma 4 (Posterior Beliefs, Decreased Outputs)** The posterior of the bank and the potential entrant, when the contract is hidden and the bank chooses first period outputs such that $q^i < q^e$ and $q^l < q^e$, is given by

$$
(p_2, p_2^e) = \begin{cases} 
(1,1), & \text{if } p_1 \in (-\infty, a - bq^e - \eta), \\
(1,\rho), & \text{if } p_1 \in [a - b\overline{q}^e - \eta, a - bq^i - \eta), \\
(\rho,\rho), & \text{if } p_1 \in [a - b\overline{q}^i - \eta, a - bq^e + \eta), \\
(\rho,0), & \text{if } p_1 \in [a - bq^e + \eta, a - bq^i + \eta), \\
(0,0), & \text{if } p_1 \in [a - bq^i + \eta, +\infty). 
\end{cases}
$$

The ex ante probabilities of the five different states in the second period are,

$$
\begin{align*}
\Pr \{ (p_2, p_2^e) = (1,1) \} &= \rho \frac{b(q^i - \overline{q}^e)}{2\eta}, \\
\Pr \{ (p_2, p_2^e) = (1,\rho) \} &= \rho \frac{b(q^e - \overline{q}^i)}{2\eta}, \\
\Pr \{ (p_2, p_2^e) = (\rho,\rho) \} &= \frac{2\eta - b(q^e - \overline{q}^i)}{2\eta}, \\
\Pr \{ (p_2, p_2^e) = (\rho,0) \} &= \frac{b(q^e - q^i)}{2\eta}, \\
\Pr \{ (p_2, p_2^e) = (0,0) \} &= (1 - \rho) \frac{b(q^i - \overline{q}^i)}{2\eta}.
\end{align*}
$$

As before, the bank’s payoff for the intervals in which the posterior of the bank and the entrant coincide are given by Equations (1), (4) and (11). Notice, however, that compared to the earlier case, now there is an interval of prices for which the entrant believes the incumbent to be
unknown while the bank believes correctly, that the incumbent is low cost, i.e., \((\rho_2, \rho_2^L) = (1, \rho)\). In this interval entry occurs even though the incumbent’s true type is low cost. Payoffs for \((\rho_2, \rho_2^L) = (1, \rho)\) are given by calculating the low costs incumbent’s best response to the entrant’s output (given in Proposition 3), yielding,

\[
u_b = \frac{(a + c_e - \frac{z + 3c}{2})^2}{9b}.
\] (14)

There is also a second interval of prices for which the beliefs diverge. Namely, the entrant believes the incumbent to be high cost while the bank believes, correctly, that the incumbent is low cost with probability \(\rho\), i.e., \((\rho_2, \rho_2^L) = (\rho, 0)\). An implication of the Corollary is that the bank’s payoff is the same as in the case \((\rho_2, \rho_2^L) = (\rho, \rho)\), so that the bank’s payoffs for \((\rho_2, \rho_2^L) = (\rho, 0)\) are given by Equation (11), i.e.,

\[
u_b = \frac{(a - \bar{c})^2}{4b} + \frac{\rho}{1 - \rho} \frac{(\bar{\tau} - \bar{c})^2}{4b}.
\] (15)

4 The First Period

In the first period, the bank maximizes its first period expected repayments plus the expected second period payoff, i.e., \(\rho R + (1 - \rho) \bar{R} + \delta EU_b\), subject to individual rationality and incentive compatibility constraints. We show below that these constraints remain unaffected by the unobservability of the contract. However, when the first period contract is hidden, the bank accounts for the possibility that the potential entrant’s beliefs differ from those of the the bank. These differing beliefs affect
the bank’s future payoff $Eu_b$ as well as the repayments $R$ and $\bar{R}$ implied by the binding constraints.

The analysis of the previous section provides the basis for calculating the bank’s second period expected payoffs for the two possibilities of differences in the belief functions. For the case that the bank considers increasing the outputs from what the entrant conjectures, Lemma 1 in conjunction with Equations (1), (4), (11), and (12) and (13) yield,

$$Eu_b^{h+} = (a - \overline{c})^2 \rho \frac{b(q^1 - \overline{q}^l)}{2\eta} + (a + c_e - 2\overline{c})^2 \frac{b(q_e - \overline{q}^l)}{2\eta} + \frac{(a + c_e - 2\overline{c})^2}{9b} \frac{\rho}{1 - \rho} \frac{(\overline{c} - c_e)^2}{4b} 2\eta - b(q^1 - \overline{q}^e) + \frac{(a - \overline{c})^2}{4b} \frac{\rho}{1 - \rho} \frac{(\overline{c} - c_e)^2}{4b} b(q_e - \overline{q}^e) + \frac{(a + c_e - 2\overline{c})^2}{9b} \frac{\rho}{1 - \rho} b(q^1 - q^e)}{2\eta} + \frac{(a + c_e - 2\overline{c})^2}{9b} \frac{\rho}{1 - \rho} \frac{(\overline{c} - c_e)^2}{4b} 2\eta - b(q^1 - \overline{q}^e),$$

(16)

Here the superscript 'h' denotes that the contract is 'hidden' as opposed to being observable, and the '+' represents the case in which the bank considers an increase of the outputs from the entrant’s conjectures, i.e., $\overline{q}^i > \overline{q}^e$ and $q^i > q^e$.

Similarly, Lemma 4 and Equations (1), (4), (11), and (14) and (15), yield,

$$Eu_b^{h-} = (a - \overline{c})^2 \rho \frac{b(q^1 - \overline{q}^l)}{2\eta} + (a + c_e - 2\overline{c})^2 \frac{b(q^1 - \overline{q}^l)}{2\eta} + \frac{(a + c_e - 2\overline{c})^2}{9b} \frac{\rho}{1 - \rho} \frac{(\overline{c} - c_e)^2}{4b} 2\eta - b(q^1 - \overline{q}^e) + \frac{(a + c_e - 2\overline{c})^2}{9b} \frac{(\overline{c} + 3c_e - 2\overline{c})^2}{2\eta} - \rho b(q^1 - \overline{q}^l) + \frac{(a + c_e - 2\overline{c})^2}{9b} \frac{\rho}{1 - \rho} \frac{(\overline{c} - c_e)^2}{4b} b(q^1 - \overline{q}^e) + \frac{(a + c_e - 2\overline{c})^2}{9b} \frac{\rho}{1 - \rho} \frac{(\overline{c} - c_e)^2}{4b} b(q^1 - \overline{q}^e)}{2\eta},$$

(17)
where the ‘−’ denotes the case in which the bank considers decreasing the first period outputs from the entrant’s conjectures, i.e., $\overline{q}^i < \overline{q}^e$ and $q^i < q^e$.

In contrast, if the first-period contract is public, the bank’s second period expected payoff, using Lemma 2 and (only) Equations (1), (4) and (11), is,

$$ Eu_b = \frac{(a - c)^2}{4b}\rho\frac{b(q - \overline{q})}{2\eta} + \frac{(a + c_e - 2\overline{c})^2}{9b}(1 - \rho)\frac{b(q - \overline{q})}{2\eta} + $$

$$ + \left(\frac{(a + c_e - 2\overline{c})^2}{9b} + \frac{\rho}{1 - \rho}\frac{(\overline{c} - c)^2}{4b}\right)\frac{2\eta - b(q - \overline{q})}{2\eta}. \quad (18) $$

The first three terms in Equations (16) and (17) are similar to those from the case of observable contracts, given in Equation (18). The fourth and fifth terms in Equations (16) and (17) contain the payoffs that the bank obtains when its beliefs differ from those of the entrant. Specifically, the fourth term in Equation (16) reflects the payoff when the entrant stays out of the market because the entrant believes the incumbent to have low cost, even though the bank is unable to infer the incumbent’s type. And the fifth term in Equation (16) includes the bank’s payoff when the entrant remains uncertain although the bank has inferred that the incumbent has high cost. Similarly, the fourth term in Equation (17) contains the payoff when the entrant enters although the bank has correctly inferred that the incumbent has low costs. Finally, the fifth term in Equation (17) gives the payoff when the entrant believes the incumbent to have high costs, yet the bank is unsure of the incumbent’s type.

All five terms in Equations (16) and (17) are multiplied by their re-
pective probabilities. Consequently, when the entrant accurately infers the first period outputs, so that for both types of incumbent $q_e = q^i$, for given first period outputs, the first three terms in Equations (16) and (17) are identical to those in the observable contract given in Equation (18), whereas the final two terms vanish.

Consolidating terms and writing the expected payoff as a function of the incumbent’s actual equilibrium outputs, $q^i$ and $\bar{q}^i$ (which the bank can determine), as opposed to those conjectured by the entrant, $q_e$ and $\bar{q}^e$ (over which the bank has no influence), yields,

$$
Eu_{b}^{h+} = \rho \left( \frac{(a - \bar{c})^2}{4} - \frac{(a + c_e - 2\bar{c})^2}{9} - \frac{(\bar{c} - \bar{c})^2}{(1 - \rho)^4} \right) \frac{q^i - \bar{q}^i}{2\eta} + 
+ \left( \frac{(a - \bar{c})^2}{4} - \frac{(a + c_e - 2\bar{c})^2}{9} \right) \frac{\bar{q}^i}{2\eta} + A',
$$

(19)

and

$$
Eu_{b}^{h-} = \rho \left( \frac{(a - \bar{c})^2}{4} - \frac{(a + c_e - 2\bar{c})^2}{9} - \frac{(\bar{c} - \bar{c})^2}{(1 - \rho)^4} \right) \frac{q^i - \bar{q}^i}{2\eta} + 
+ \rho \left( \frac{(a - \bar{c})^2}{4} - \frac{(a + c_e - \frac{\bar{c} + 3c}{2})^2}{9} \right) \frac{\bar{q}^i}{2\eta} + A'',
$$

(20)

where $A'$ and $A''$ are functions of the $q^e$s, but do not depend on the $q^i$s. In contrast, the expected second period profits of the bank in the public contract case are,

$$
Eu_b = \rho \left( \frac{(a - \bar{c})^2}{4} - \frac{(a + c_e - 2\bar{c})^2}{9} - \frac{(\bar{c} - \bar{c})^2}{(1 - \rho)^4} \right) \frac{q - \bar{q}}{2\eta} + A,
$$

(21)

where $A$ does not depend on $\bar{q}$ or $\bar{q}$. Notice that there is an additional term involving the high-cost incumbent’s first-period output in the hid-
den contract case, Equations (19) and (20), compared to the observable contract case, Equation (21).

Having thus determined the bank’s future expected payoff when it accounts for possible differences in beliefs, consider now the constraints of the two types of incumbent. As in the static contract, the high cost incumbent’s individual rationality constraint bounds and the low cost incumbent’s incentive compatibility constraint binds.\(^{10}\) Since the high cost incumbent’s expected payoff is zero regardless of the bank’s or the entrant’s beliefs, the individual rationality constraint for the hidden and the public contract are the same as in the static problem,

\[ R = (a - b\bar{q} - \tau)\bar{q}. \] (22)

The low-cost incumbent’s incentive compatibility constraint differs from the static second period constraint in that the low cost firm must be paid up-front its discounted potential gain from deception.\(^{11}\) Specifically, if the low cost firm mimics the high cost firm and produces \(q_i\) in the first period, then for sufficiently large \(\epsilon\) the bank believes that the firm has high cost. Consequently, the bank offers a second period contract

\(^{10}\)An implication of the possible uncertainty in the second period guaranteed by Assumption 2 is that the incentive compatibility constraint of the high cost firm does not bind in the first period. That is, the analogue to the “take-the-money-and-run strategy” is not of concern. On this, see JMS.

\(^{11}\)Despite the possibility of signal jamming, in what follows, one need only consider the case in which the bank is fully deceived, i.e., \(\rho_2 = 0\), because the incumbent’s payoff for the case where \(\rho_2 = \rho\) is the same whether or not deception was attempted, and the payoff for the case where \(\rho_2 = 1\) is always 0—regardless of what the entrant’s posterior \(\rho^*_2\) is.
with the high cost firm in mind. Due to the low cost incumbent’s cost advantage, the low cost firm accepting this contract in the second period obtains a positive payoff. Specifically, whenever the bank is deceived so that $\rho_2 = 0$, the entrant enters, since $\rho_2^e \in \{0, \rho\}$, (see Lemmas 1 and 4), and in these cases a low cost firm obtains a payoff of $(\bar{c} - c)\bar{q}_t$. Taking into consideration the probability that $\epsilon$ is large enough for such a deception to occur and inserting the appropriate value for $\bar{q}_t$, the incentive compatibility constraint of the incumbent firm with low cost yields,

$$R = (a - bq - c)q - (\bar{c} - c)\bar{q} - \delta(\bar{c} - c) \left( \frac{a + c_0 - 2\bar{c}}{3b} \right) \frac{b(q - \bar{q})}{2\eta}.$$  \(23\)

The first two terms on the right hand side are analogous to those obtained in the static second period constraint, given in Equation (9). The third term is the amount by which the low cost incumbent’s repayment is reduced in order to induce him to choose the output that may reveal his type.

Again, when successfully deceiving the bank so that the bank’s posterior $\rho_2 = 0$, the incumbent’s second period payoff is the same regardless of whether the entrant believes to have inferred the incumbent’s type or not. This is due to the result that the entrant’s output decision is independent of his beliefs, once entry has occurred, see the Corollary. Consequently the low cost firm’s incentive compatibility constraint remains unaffected by the unobservability of the contract.

The bank’s objective is to choose the first period outputs and ex-
pected repayments to maximize,

\[ \rho R + (1 - \rho)\overline{R} + \delta Eu^h, \quad \text{subject to (22) and (23)}, \]

where \( Eu^h \) is given in Equations (19) and (20). The first period equilibrium outputs, denoted by \( q_h \) and \( \overline{q}_h \), which are implied by the solution to this problem, are given in the following theorem.

**Theorem 1 (Hidden Contract)** When the first period contract between the bank and the incumbent firm is hidden, any pair of outputs that satisfy the following conditions can be supported in equilibrium.

\[ q_h = \frac{a-c}{2b} + \frac{\delta}{2b} B \frac{1}{2\eta}, \]

\[ \overline{q}_h \in [\overline{q}_{h+}, \overline{q}_{h-}], \quad \text{where} \]

\[ \overline{q}_{h+} = \frac{a-c}{2b} - \frac{\rho}{1-\rho} \left( \frac{(a-c)^2}{2b^2} + \frac{\delta}{2b} \left( B - \frac{1}{\rho} \left( \frac{(a-c)^2}{4} - \frac{(a+c_e-2c)^2}{9} \right) \right) \right) \frac{1}{2\eta}, \]

\[ \overline{q}_{h-} = \frac{a-c}{2b} - \frac{\rho}{1-\rho} \left( \frac{(a-c)^2}{2b^2} + \frac{\delta}{2b} \left( B - \frac{1}{\rho} \left( \frac{(a-c)^2}{4} - \frac{(a+c_e-2c)^2}{9} \right) \right) \right) \frac{1}{2\eta}, \]

and \( B = \left( \frac{(a-c)^2}{4} - \frac{(a+c_e-2c)(a+c_e-3c_e+c)}{9} - \frac{(a-c)^2}{(1-\rho)^4} \right). \)

**Proof** The first order conditions of the maximization program yield \( q_h \). To determine \( \overline{q}_h \), notice that for given \( \overline{q}^e \) the right-derivative of the objective function is determined by using (19) in the objective function, whereas the left-derivative is determined using (20). For any \( \overline{q}^e \in [\overline{q}_{h+}, \overline{q}_{h-}] \) the left-derivative is positive and the right-derivative is negative, so that the objective function obtains a maximum at \( \overline{q}^e \). Any \( \overline{q}^e \notin [\overline{q}_{h+}, \overline{q}_{h-}] \) cannot be part of a rational expectations equilibrium,
because the bank’s objective function increases when choosing outputs that are located between the entrant’s conjecture and the interval of equilibrium outputs.

\[ Q.E.D.\]

The derivative of the bank’s first period objective function is illustrated in Figure 2. Also indicated is an equilibrium for a particular value of \( \bar{q}_h = \bar{q}^e \) from the interior of the interval.

---Figure 2 about here---

**Figure 2**: Generic left- and right-derivatives stemming from Equations (19) and (20) (thin continuous lines), as well as the actually relevant first derivative of the objective function for the equilibrium level of \( \bar{q}_h = \bar{q}^e \) depicted (thick discontinuous line).

**Theorem 2 (Existence)** There exists a non-empty continuum of equilibrium hidden contracts, whenever

\[
\rho > \left( \frac{(a - \bar{c})^2}{4} - \frac{(a + c_e - 2\bar{c})^2}{9} \right) / \left( \frac{(a - \bar{c})^2}{4} - \frac{(a + c_e - \tau + 3\bar{c})^2}{9} \right).
\]

Otherwise no rational expectations separating equilibrium exists.

**Proof** It is straightforward to show that the interval \([\bar{q}_{h+, \bar{q}_{h-}}]\) has positive length \( \text{iff} \) the condition on \( \rho \) is met. \[ Q.E.D. \]
The two differences in the numerator and denominator in the condition on $\rho$ in Theorem 2 measure changes in profit when going from Cournot competition to monopoly, whereas the difference between the numerator and the denominator approximates this change in profits as cost decreases from $\bar{c}$ to $c$.

Figure 2 also demonstrates the non-existence of the equilibrium when the condition on $\rho$ is not met. As the two generic derivatives change places there is no consistent (rational expectations) output for the bank’s choice of output and the entrant’s beliefs about this output.

Signal jamming impacts the bank’s choice of first period output levels. In particular,

**Theorem 3 (Signal Jamming)** Only the high cost incumbent’s first period output is affected by whether the contract is hidden or observable. That is,

$$q_h = \bar{q},$$

$$\bar{q}_h = \bar{q}_h^* = \bar{q} + \frac{1}{1-\rho} \frac{\delta}{2b} \left( \frac{(a-\bar{c})^2}{4} - \frac{(a+c_2-2\bar{c})^2}{9} \right) \frac{1}{2\eta} > \bar{q}.$$

Here $\bar{q}$ and $q$ are the equilibrium outputs under the public contract.

**Proof** The proof follows immediately from a comparison of the equilibrium first-period outputs given in Theorem 1 above and the corresponding outputs for the public contract case given below. The outputs for the public contract are the solutions to the bank’s maximization problem with $\rho R + (1-\rho)R + \delta E u_b$ as the objective function (where $E u_b$ is given in Equation (21)) and Equations (22) and (23) as the constraints. These
are given by,

\[
q = \frac{a-c}{2b} + \frac{\delta}{2b} B \frac{1}{2\eta},
\]
\[
\bar{q} = \frac{a-c}{2b} - \frac{\rho}{1+\rho} \left( \frac{a-c}{2b} + \frac{\delta}{2b} B \frac{1}{2\eta} \right),
\]

where B is given in Theorem 1. \(Q.E.D.\)

Theorem 3 says that the low cost incumbent’s output remains unchanged by signal jamming whereas the high cost incumbent’s output increases. To see this result intuitively, consider the impact of the magnitude of \(q\) on the bank’s future expected payoffs for the case of the observable contract without signal jamming (given in Equation (18)) and of \(q^i\) in the case of a hidden contract with signal jamming when the bank increases the first period outputs (given in Equation (16)) from the outputs postulated by the entrant. The low cost incumbent’s first period outputs \((q\) and \(q^i\), respectively) affect the bank’s future payoff by the same amount in the case that both the bank and potential entrant infer that the incumbent has low costs (the first term in both (16) and (18)), as well as in the case that both bank and potential entrant remain ignorant of the incumbent’s costs (the third terms in both equations).

Now consider the second terms in (16) and (18). These are the bank’s second period expected payoffs when the bank and potential entrant both believe that the incumbent has high costs. When the contract is observable so that there is no signal jamming, this term is affected by the low cost incumbent’s first period output \(q\), because this output affects the probability that both bank and potential entrant believe the incumbent
has high costs. On the other hand, when the contract is hidden so that there is signal jamming, the low cost incumbent’s first period output $q^i$ does not affect the probability that both the bank and the potential entrant believe that the incumbent has high costs (and hence does not affect the second term in (16)), because the potential entrant’s beliefs are determined by $q^e$ and not the unobservable $q^i$.

However, in the case of signal jamming, the unobservable $q^i$ affects the probability that the potential entrant remains unsure about the incumbent’s costs, while the bank properly infers that the incumbent has high costs (see the fifth term in (16)). Now notice that, due to Proposition 1 and its Corollary, the bank’s expected second period payoff is the same for the case of no signal jamming with posterior of $\rho_2 = \rho_2^e = 0$ and the case of signal jamming with beliefs of $(\rho_2, \rho_2^e) = (0, \rho)$.

Consequently, the marginal impact on the bank’s future expected payoff of changes in the low cost incumbent’s first period output when contracts are observable and there is no signal jamming, i.e., the marginal impact of $q$ (determined by the first, second, and third terms in Equation (18)) is identical to the marginal impact of changes when the contract is hidden so that signal jamming occurs, i.e., changes in $q^i$ (determined by the first, fifth, and third terms in Equation (16)). Hence, after consolidating the bank’s future expected payoffs in the two cases, Equations (16) and (18), the impact of the low cost incumbent’s output ($q$ and $q^i$, respectively) under the different contracts are the same.
The argument for when the bank considers a decrease in the first period outputs, runs along the same lines. Now, however, the case of \( (\rho_2, \rho_2^c) = (\rho, 0) \) takes the place of \( (\rho_2, \rho_2^c) = (0, \rho) \). Nevertheless, after consolidating the bank’s future expected payoff (now Equation (17)), the impact of the low cost incumbent's output \( q^i \) is the same as in Equation (18).

Consider now the second statement in Theorem 3, namely the comparison of the high cost incumbent’s first period outputs for the two cases, \( \overline{q} \) from Equation (24) and \( \overline{q}_h \) from Theorem 1. As in the argument made concerning the low cost incumbent’s first period output, the high cost incumbent’s output affects the bank’s future payoffs through the probabilities that the incumbent’s type is revealed. Thus, for the case that the bank increases the first period outputs from those postulated by the entrant, the first two terms in Equations (16) and (18) are the same. Moreover, in the case of the hidden contract and signal jamming, the bank’s choice of \( \overline{q}^i \) does not affect the probability that the potential entrant remains ignorant of the incumbent’s cost. That is, the third term in (16) depends only on the potential entrant’s conjecture about the high cost incumbent’s output \( \overline{q}^e \), not on the actual output \( \overline{q}^i \). However, the choice of \( \overline{q}^i \) affects the probability that the potential entrant believes the incumbent firm to have low cost, when in fact the price does not allow this inference. That is, the choice of \( \overline{q}^i \) affects the fourth term in (16).

Now notice that in affecting the probability that the potential entrant
believes that the incumbent has low costs, the bank changes the probability of entry. This change in the probability of entry yields an economically significantly different second period, since it follows a different first period contract. That is, the implications of Proposition 3 and its Corollary do not apply when considering the bank’s choice of $\bar{q}^{t}$. Consequently, $\bar{q}_{h} > \bar{q}$.

The case when the bank decreases the first period outputs follows along the same lines. Now, however, entry may occur, even when the bank has inferred that the incumbent is low cost. While this accounts for the difference between Equations (16) and (17), it still yields that $\bar{q}_{h} > \bar{q}$.

The following result is useful in providing intuition for why the bank chooses to set the first-period output of the high cost incumbent higher with signal jamming than without. It is also convenient for determining the effect on the probability of entry later.

**Proposition 4** For any given conjecture about the first period outputs, $\bar{q}^{e}$ and $\underline{q}^{e}$, the probability of entry is decreasing in the actual equilibrium levels of output, $\bar{q}^{i}$ and $\underline{q}^{i}$. Formally,

$$\frac{d}{d q^{i}} \Pr \{ \text{entry} \mid \bar{q}^{e}, q^{e} \} \leq 0, \quad \forall q^{i} \in \{ \bar{q}^{i}, \underline{q}^{i} \}.$$ 

**Proof** Recall from Lemma 3 that the potential entrant stays out of the market whenever, $\rho_{2}^{e} = 1$, and enters otherwise. Given the potential entrant’s conjecture that the first period induced outputs are given by $\bar{q}^{e}$ and $\underline{q}^{e}$, the entry rule in conjunction with the belief function and
objective posterior probabilities of second period beliefs, yield,

$$\Pr \{ \text{entry} \mid q^e, q^e \} = \Pr \{ p_1 > a - b\bar{q}^e - \eta \mid q^e, q^e \}$$

$$= \Pr \{ a - bq^i + \epsilon > a - b\bar{q}^e - \eta \mid \bar{q}^e \}$$

$$= \max \left\{ 0, \rho \frac{2\eta - b(q^i - \bar{q}^e)}{2\eta} \right\} + \max \left\{ 0, (1 - \rho) \frac{2\eta - b(\bar{q} - \bar{q}^e)}{2\eta} \right\},$$

and the result follows. \(Q.E.D.\)

This result reflects the importance of the unobservability of the financial contract. If the contract were public, the probability of entry is increasing in \(q^i\). Thus, increasing \(q^i\), when the contract is public, increases the probability of entry in the second period and hence reduces the bank’s payoffs. When the contract is unobservable however, increasing \(q^i\), for given \(q^e\), decreases the probability of entry and thus increases payoffs. This result, thus, provides some intuition for why the equilibrium first-period output of the high cost incumbent is set higher with unobservable contracts in contrast to the public contract case.

An implication of the changes of the equilibrium outputs in the first period, due to signal jamming, is that when the contract is hidden the probability that the potential entrant enters the market is actually increased. Formally,

**PROPOSITION 5** *The probability of entry is greater when the contract is hidden than when it is observable. That is, signal jamming leads to increased probability of entry, or,*

$$\Pr \{ \text{entry} \mid q_h \} > \Pr \{ \text{entry} \mid q \}.$$
PROOF Substituting $q_h$s for the $q^i$s and $q^e$s in the proof of Proposition 4 yields $\Pr\{|\text{entry}|q_h\}$, and an equivalent substitution by $q$ yields $\Pr\{|\text{entry}|q\}$. Specifically,

$$\Pr\{|\text{entry}|q_h\} = \rho \frac{2\eta - b(q_h - \bar{q}_h)}{2\eta} > \rho \frac{2\eta - b(q - \bar{q})}{2\eta} = \Pr\{|\text{entry}|q\},$$

since by Theorem 3, $q_h = q$ and $\bar{q}_h > \bar{q}$. \hspace{1cm} Q.E.D.

The reason that entry is more likely when the contract is hidden than when it is observable is an immediate implication of Theorem 3. Simply put, signal jamming decreases the distance between the two equilibrium first period output levels $\bar{q}_h$ and $q_h$ compared to the case of observable contracts. Underlying this is the bank’s attempt to manipulate the probability of entry. Specifically, in the attempt to decrease the probability of entry, the bank increases the high cost incumbent’s first period output $\bar{Q}_i$ (while all else remains the same). However, in equilibrium, the potential entrant anticipates this attempt at manipulating the probability of entry, so that $\bar{q}^e = \bar{q}_i^l$. Notice that the probability of entry, as derived in the proof to Proposition 4, is—all else being equal—increasing in $\bar{q}^e$.

The intuition for the result is straightforward. When the first period contract is observable, the contract serves as a public commitment to the specified levels of output. When the contract is private (i.e., hidden) and not observable, no such commitment exists, so that the bank attempts to use the contract to manipulate the probability of entry. Since the bank cannot control the the potential entrant’s beliefs, the only way the
probability of entry can be affected is by manipulating the first period outputs. Such manipulation is obviously costly in the short run (in the first period). However, it also does not pay in the long run (in the second period), because in equilibrium, such manipulation is anticipated by the potential entrant and hence thwarted.

There is a close analogy to the models of MR and JJM2. In MR, the incumbent firm has no method to commit to not signal jam (i.e., to not limit price) and hence engages in costly deviations from myopic pricing policies, yet any potential entrant is aware of limit pricing so that the probability of entry is in fact not affected. In contrast to this, the contract between the bank and the incumbent firm in JJM2 serves as the necessary commitment device that allows optimal first period pricing to be credible.

To see why, notice that, in general, the bank can expect to obtain a higher second period payoff if there is no entry, simply because the incumbent firm generates higher profit when it operates as a monopolist in the second period than when it faces competition. Consequently, the bank would like to adjust the contracts with the incumbent firm in an attempt to reduce the probability of entry. Clearly, given any conjecture of the potential entrant about the first period outputs, the bank is unable to affect the entrant’s entry rule, Lemma 3. That is, the bank is not able to trigger a different entry decision, given an observed price. However, the bank can affect the probability of different observed prices as the equilibrium outputs implied by the contract determine the distribution
of observed prices and thus affect the probability of entry.

That is, by manipulating the first period (hidden) contract, the bank affects the actual outputs that the two types of incumbent produce. This, in turn, affects the probability distribution of the first period price and may lead the potential entrant to refrain from entry on the basis of his belief that the incumbent has low cost, even though the bank is aware that the observed price does not in fact allow this inference. Such an attempt to manipulate the second period game by choice of the first period contract gives rise to signal jamming.

**Signal Dampening and Strategic Experimentation**  Finally, we discuss how the threat of entry with a hidden contract affects the signal dampening and the strategic experimentation effects discussed in JJM1 in the public contract regime. Recall that in the dynamic agency model of JM and JMS, the first period outputs change to balance the incentive to increase the bank’s first period payoff by keeping the outputs close together (the signal dampening effect) and the incentive to increase the expected second-period payoff by setting outputs further apart (the experimentation effect). JMS show that when the noise is uniformly distributed, the signal dampening effect dominates and the first-period outputs are set closer together in equilibrium. JJM1 show that when the financial contract is public, the threat of entry weakens the signal dampening effect and strengthens the strategic experimentation effect and thus, overall, the first period outputs are set further apart with entry than without. An
implication of this result is that the bank learns more with the threat of entry than without.

In this paper, we have shown in Theorem 3 that the incumbent's outputs in the first period are set closer together than when the contract is public. The underlying analysis shows that unobservability of the contract does not affect signal dampening, that is, the low cost incumbent's incentive to mimic the high cost incumbent is unchanged by the fact that the entrant does not observe the financial contract. This is because the benefit from mimicking occurs in the state when the bank believes the incumbent to be high-cost when he actually is low-cost, and the payoff from mimicking is the same as in the public contract case.

However, the strategic experimentation effect is changed due to the possibility of signal jamming. In particular, the incentive of the bank to learn decreases, because of the need to benefit from the entrant's conjectures. In other words the bank can no longer manipulate the entrant's beliefs, as in the public contract case. Hence, the bank's attempt to manipulate the entrant is not successful and leads to a greater probability of entry. That is, the expected second period payoffs are lower compared to the public contract case and, thus, the bank chooses to set the outputs closer together, which in turn leads to a higher probability of entry.

Finally, we comment on the broader issue of the effect of financial contracting on the entry game and vice-versa, when the financial contract is hidden. JJM1 show that when the contract is public, the threat
of entry leads the outputs to be further apart in the first period. This is different from the traditional notion of limit pricing. On one hand, the bank increases the low cost incumbent’s output and on the other decreases the high cost incumbent’s output. Intuitively, this effect has to do with the combined incentive of the bank to deter entry and to learn. When the contract is hidden, as is the case in this paper, the low cost incumbent’s output is the same as in JJM1 and thus is increased compared to the model without entry. However, the high cost incumbent’s output is increased compared to JJM1 but may or may not be higher than the output without entry. Thus, the direction of the change in the high cost incumbent’s output indicates a result closer to traditional limit pricing with hidden contracts rather than public contracts. As for the effect of the threat of entry on the financial contract, JJM1 show that for given outputs, the low cost incumbent’s repayment increases. The same is true here since the incentive compatibility constraint does not change.

References


