Appendix to “Deriving the wage-wage and price-price Phillips curves from a model with efficiency wages and imperfect information”

An expression for $\hat{L}_t$ can be obtained by totally differentiating (14) and dividing by the original equation, yielding

$$\hat{L}_t = \frac{\gamma \hat{W}_t - \hat{Y}_t - \phi(\gamma - 1)\hat{A}_t - \phi(\gamma - 1)e^{-1}[e_w \frac{W_t}{P_t} \hat{W}_t - e_w \frac{W_t}{P_t} \hat{P}^e + e_u du_t] - \gamma \hat{P}_t}{\phi(\gamma - 1) - \gamma}.$$ 

By making the substitutions $du_t = -s_L \hat{L}_t + s_L \psi \hat{W}_t - s_L \psi \hat{P}_t^e$, $ee^{-1} = W_t / \hat{P}_t^e$ (from (15)), and $\hat{P}_t = \hat{M}_t - \hat{Y}_t$, the solution for $\hat{L}_t$ becomes

$$\hat{L}_t = \frac{[\phi(\gamma - 1) - \gamma + \phi(\gamma - 1)e^{-1}e, s_L \psi] \hat{W}_t - \phi(\gamma - 1)[1 + e^{-1}e, s_L \psi] \hat{P}_t^e}{(\gamma - 1)\hat{Y}_t + \phi(\gamma - 1)\hat{A}_t + \gamma \hat{M}_t}.$$ 

(A1)

Since each firm’s output ($Q$) equals aggregate demand per firm ($Y$) in a representative firm framework, an equation for $\hat{Y}_t$ can be obtained by differentiating (13) and making the substitutions $du_t = -s_L \hat{L}_t + s_L \psi \hat{W}_t - s_L \psi \hat{P}_t^e$ and $ee^{-1} = W_t / \hat{P}_t^e$ . Accordingly,

$$\hat{Y}_t = \phi \hat{A}_t + \phi \hat{L}_t + \phi \hat{W}_t - \phi \hat{P}_t^e - \phi e^{-1}e, s_L \hat{L}_t + \phi e^{-1}e, s_L \psi \hat{W}_t - \phi e^{-1}e, s_L \psi \hat{P}_t^e.$$ 

(A2)

Substituting (A2) into (A1) yields $\hat{L}_t = \hat{M}_t - \hat{W}_t$. Thus, the unemployment rate can be expressed as

$$du_t = -s_L (\hat{M}_t - \hat{W}_t) + s_L \psi \hat{W}_t - s_L \psi \hat{P}_t^e.$$ 

(A3)
By substituting (A3) into (16), the following equation is obtained:

\[
\begin{bmatrix}
1 - s_L \frac{e_u - e_{wu}}{e_{ww}} (1 + \psi)
\end{bmatrix} \hat{W}_t = \begin{bmatrix}
1 - s_L \psi \frac{e_u - e_{wu}}{e_{ww}} \\
\end{bmatrix} \hat{P}_t^{\epsilon} - s_L \frac{e_u - e_{wu}}{e_{ww}} \hat{M}_t,
\]

which can be expressed as

\[
\hat{W}_t = z \hat{P}_t^{\epsilon} + (1 - z) \hat{M}_t, \tag{A4}
\]

where \( z = \frac{e_{ww} - s_L \psi (e_u - e_{wu})}{e_{ww} - s_L (e_u - e_{wu})(1 + \psi)} \).

By substituting \( \hat{L}_t = \hat{M}_t - \hat{W}_t \) into (A2) and then substituting the resulting expression for \( \hat{Y}_t \) into the equation, \( \hat{P}_t = \hat{M}_t - \hat{Y}_t \), the following expression for the price level is derived:

\[
\hat{P}_t = [1 - \phi + \phi e^{-1} e_u s_L] \hat{M}_t - \phi \hat{A}_t - \phi e^{-1} e_u s_L (1 + \psi) \hat{W}_t + \phi (1 + e^{-1} e_u s_L \psi) \hat{P}_t^{\epsilon}. \tag{A5}
\]

The variable \( \hat{W}_t \) can be eliminated from (A5) by substituting (A4) into (A5), which yields

\[
\hat{P}_t = [1 - \phi + \phi e^{-1} e_u s_L (z + \psi z - \psi)] \hat{M}_t - \phi \hat{A}_t + [\phi - \phi e^{-1} e_u s_L (z + \psi z - \psi)] \hat{P}_t^{\epsilon}. \tag{A6}
\]

In addition, the following equation for \( \hat{M}_t \) is obtained by substituting (A4) into (A3):

\[
\hat{M}_t = \frac{1}{s_L (z + \psi z - \psi)} du_t. \tag{A7}
\]

Finally, substituting (A7) into (A6) enables the price level to be expressed as

\[
\hat{P}_t = \hat{P}_t^{\epsilon} - \frac{1 - \phi + \phi e^{-1} e_u s_L (z + \psi z - \psi)}{s_L (z + \psi z - \psi)} du_t - \phi \hat{A}_t.
\]
\[ \hat{P}^e_t = \frac{(1 - \phi)(e_{WW} - s_L (e_u - e_{WW})(1 + \psi)) + \phi e^{-1} e_u s_L e_{WW}}{s_L e_{WW}} du_t - \phi \hat{A}_t. \] (A8)