Theory

Lots of things vibrate or oscillate. A vibrating tuning fork, atoms within molecules, and the loudspeaker in a radio are all examples of physical vibrations.

One simple system that vibrates is a mass hanging from a spring. Imagine a spring that is hanging vertically from a support. When no mass hangs at the end of the spring, it has a length $L$ (called its rest or equilibrium length). When a mass is added to the spring, its length increases by $\Delta L$. The equilibrium position of the mass is now a distance $L + \Delta L$ from the springs support. What happens when the mass is pulled down a small distance from the equilibrium position? The spring exerts a restoring force, $F = -kx$, where $x$ is the distance the spring is pulled down and $k$ is the force constant of the spring (also called the `spring constant'). The negative sign indicates that the force points opposite to the direction of the displacement of the mass. The restoring force causes the mass to oscillate up and down. The period of oscillation depends on the mass and the spring constant.

When the restoring force of the spring obeys Hooke’s law (when the distance the spring is stretched is not too large), simple harmonic motion results.

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Two waves which differ in their phase. In this case, they differ in their phase by 1/4. As such, the first wave is nothing but the mathematical plot of the Cosine function, whereas the second wave is the plot of the Sine function. As can be seen, phase depends only upon what instant we choose t = 0 to be.
Energy is present in three forms for the mass and spring system. The mass $m$, with velocity $v$, can have kinetic energy $KE$

$$KE = \frac{1}{2}mv^2$$

The spring can hold elastic potential energy, or $PE_{\text{elastic}}$. We calculate $PE_{\text{elastic}}$ by using

$$PE_{\text{elastic}} = \frac{1}{2}ky^2$$

where $k$ is the spring constant and $y$ is the extension or compression of the spring measured from the equilibrium position.

The mass and spring system also has gravitational potential energy ($PE_{\text{gravitational}} = mgy$), but we do not have to include the gravitational potential energy term if we measure the spring length from the hanging equilibrium position. We can then concentrate on the exchange of energy between kinetic energy and elastic potential energy.

If there are no other forces experienced by the system, then the principle of conservation of energy tells us that the sum $\Delta KE + \Delta PE_{\text{elastic}} = 0$, which we can test experimentally.

Objectives

- Measure and analyze the energy of an oscillating mass and spring system.
- Determine if conservation of energy applies to this SHM system
### Apparatus

<table>
<thead>
<tr>
<th>Windows PC</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>LabPro</td>
<td>Tapered SHM spring (small end at top)</td>
</tr>
<tr>
<td>Vernier Motion Detector 2</td>
<td>Long vertical stand/ horizontal rod, clamps</td>
</tr>
<tr>
<td>50, 100, 150 and 200g masses</td>
<td>Tape</td>
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** SETUP Also, use a long enough horizontal rod so the mass/ spring are at least 20 cm from the edge of the desk. ** Don't put the computer monitor close to the SHM setup since it can interfere with the motion detector.
SHM - Preliminary Observations – Take 10 minutes and work with your lab group

1. Sketch a graph of the height vs. time for the mass on the spring as it oscillates up and down through one cycle. Mark on the graph the times where the mass moves the fastest and therefore has the greatest kinetic energy. Also mark the times when it moves most slowly and has the least kinetic energy.

2. On your sketch, label the times when the spring has its greatest elastic potential energy. Then mark the times when it has the least elastic potential energy.

3. Sketch graphs of kinetic energy vs. time and elastic potential energy vs. time.

Your TA will lead a group discussion at this point.

Data Collection

1. Set up the experiment.
   a. Mount and TAPE the 100 g mass on the hanger and attach them to the spring as shown in Figure 1.
   b. Connect the Motion Detector to the DIG/SONIC 1 channel of the interface. If the Motion Detector has a switch, set it to Normal (rightmost).
   c. Position the Motion Detector directly below the hanging mass, taking care that no extraneous objects could send echoes back to the detector. Protect the Motion Detector by placing a wire basket over the detector. The mass should be about 30 cm above the detector when it is at rest. Using amplitudes of 10 cm or less will then keep the mass outside of the 15 cm minimum distance of the Motion Detector.

2. Open the file “17a Energy in SHM” from the Physics with Vernier folder.

3. Start the mass moving up and down by lifting it 8 cm and then releasing it. Take care that the mass is not swinging from side to side. Click Collect to record position and velocity data. Review your graphs and compare to your predictions. Comment on any differences.

4. To calculate the spring potential energy, it is necessary to measure the spring constant k. Hooke’s law states that the spring force is proportional to its extension from equilibrium, or \( F = -kx \). You can apply a known force to the spring, to be balanced in magnitude by the spring force, by hanging a range of weights from the spring. The Motion Detector can then be used to measure the equilibrium position. Open the experiment file “17b Energy in SHM.” Logger Pro is now set up to plot the applied weight vs. position.

5. Click Collect to begin data collection. Hang a 50 g mass from the spring and allow the mass to hang motionless. Click Keep and enter 0.49, the weight of the mass in newtons (N). Press ENTER to complete the entry. Now hang 100, 150, and 200 g masses from the spring, recording the position and entering the weights in N. When you are done, click Stop to end data collection.
6. Click on the Linear Fit button, $\square$, to fit a straight line to your data. The magnitude of the slope is the spring constant $k$ in N/m. Record the value in the data table below.

7. Put the 100g mass back on the hanger with tape. Open the experiment file “17c Energy in SHM.”

8. You may need to modify the “Sampling” rate – click on the “Experiment” drop down menu, then select “Data Collection” and change the “Sampling Rate” to 30. Adjust the parameter for the mass and spring constant in the lower right boxes as appropriate.

9. With the mass hanging from the spring and at rest, click $\square$ Zero to zero the Motion Detector. From now on, all distances will be measured relative to this position. When the mass moves closer to the detector, the position reported will be negative.

10. Start the mass oscillating in a vertical direction only, with an amplitude of about 8 cm. Click $\square$ Collect to gather position, velocity, and energy data. When you have a good plot of position vs. time, have your TA review it before you proceed.

**DATA TABLE**

<table>
<thead>
<tr>
<th>Spring constant</th>
<th>N/m</th>
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**ANALYSIS/ DISCUSSION**

1. Click (or right click) on the $y$-axis label of the velocity graph to choose another column for plotting. Click on More to see all of the columns. Uncheck the velocity column and select the kinetic energy and potential energy columns. Click $\square$ OK to draw the new plot.

2. Export your SHM data to a CSV file for use in preparing your lab report.

3. Compare your two energy plots to the sketches you made earlier. Be sure you compare to a single cycle beginning at the same point in the motion as your predictions. Comment on any differences.

4. If mechanical energy is conserved in this system, how should the sum of the kinetic and potential energies vary with time? Choose Draw Prediction from the Analyze menu and draw your prediction of this sum as a function of time.

5. Check your prediction. Click on the $y$-axis label of the energy graph to choose another column for plotting. Click on More and select the total energy column in addition to the other energy columns. Click $\square$ OK to draw the new plot.

6. From the shape of the total energy vs. time plot, what can you conclude about the conservation of mechanical energy in your mass and spring system?
Refer to the P150A Lab Syllabus for information on the general guidelines for writing an Experimental Lab report.

- **This week's Lab report must include the following:**

  **Your Theory section should include:**

  A. An explanation in your own words of Simple Harmonic motion and a list of examples of SHM that we have discussed in lecture or you have read about.

  B. Include the formulas and define in your own words what the potential and kinetic energies represent when a mass moves in SHM on a spring.

- **Your data section should contain:**

  The data table from Logger Pro (export as CSV) and the following excel graphs:
  - Position vs. time
  - Velocity vs. time
  - Kinetic energy and potential energy vs. time (on the same graph)

**Observation Questions:**

1. Describe how the potential energy of the mass changes when compared to the velocity of the mass.

2. What is the position of the mass when the velocity is the largest? What is the position of the mass when the velocity is the smallest?

3. What is the position of the mass when the potential energy is the largest? What is the position of the mass when the potential energy is the smallest?